Section 3

REACTIVE ELEMENTS IN POWER ELECTRONIC SYSTEMS

The conditioning of power flow in PES is done through the use electromagnetic and reactive elements (inductors, capacitors and transformers). In this section the basics of electromagnetics is reviewed. The type of capacitors popular in power electronic applications are also given. They are formulated in such a way as to be useful for the design of inductors and transformers.

ELECTROMAGNETICS

The voltage across and current through a conducting element is related through Ohm's law. This law may be stated as follows. When an electric field (of intensity ε V/m) is set up across a conducting material (of conductivity σ $1/\Omega$-m), there is an average flow of electrical charges across the conducting material (of current density J A/m). This is shown in Fig. 1.

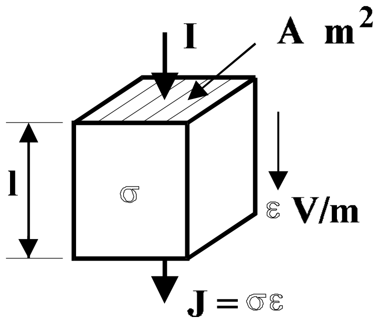


Fig. 1

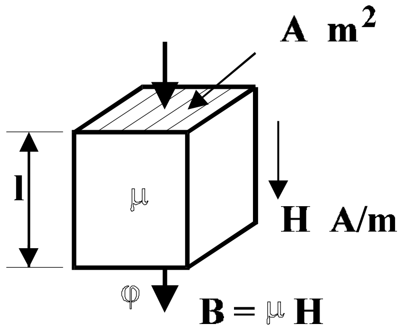


Fig. 2

$$J = \sigma \varepsilon$$

When expressed in terms of element voltage and current, this reduces to the familiar statement of Ohm's law.

$$I = \frac{V}{R} \quad ; \qquad R = \frac{l}{\sigma A}$$

In comparison with conducting materials, the property of magnetic materials may be stated as follows. When a magnetic field (of intensity H A/m) is set up, across a magnetic material (of permeability μ H/m) a magnetic flux (of density B Tesla) is established in the magnetic material, as shown in Fig. 2..

$$B = \mu H$$

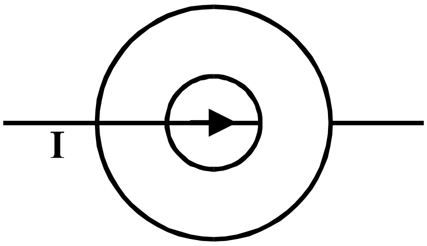
The above equation, in terms of the magnetomotive force (mmf) F and the flux Φ in the magnetic circuit, reduces to

$$\Phi = \frac{F}{R} \qquad ; \qquad R = \text{reluctance of the magnetic circuit} = \frac{l}{A\mu}$$

The above relationship is analogous to Ohm's law for magnetic circuits. The magnetic permeability μ of any magnetic material is usually expressed relative to the permeablity of free space ($\mu_O = 4\pi\, 10^{-7}$ H/m).

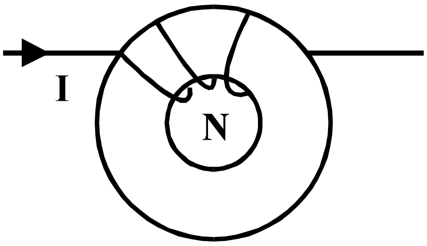
$$R = \frac{l}{A\mu_O\mu_r}$$

Electromagnetic circuit elements consist of an electric circuit and a magnetic circuit coupled to each other. The electric current in the electric circuit sets up the magnetic field in the magnetic circuit with resultant magnetic flux. Seen as an electrical circuit element, the electromagnetic element possesses the property of energy storage without dissipation.



F = I

Fig. 3a



F = NI

Fig. 3b

Ampere's law and Faraday's law relate the electric and magnetic circuits of the electromagnetic element. Ampere's law states that the mmf in a magnetic circuit is equal to the electric current enclosed by the magnetic circuit. For example for the electromagnetic circuits shown in Figs 3a and 3b, the magnetic circuit mmf's are *I* and *NI* respectively. With further assumption that the magnetic material is isotropic and homogenous and that the magnetic flux distribution is uniform. Using Ampere's law, we may relate the magnetic flux in the magnetic crcuit as

$$\Phi = \frac{\Sigma I}{R} = \frac{NI}{R}$$

The above equation may conveniently be put in the equivalent circuit shown in Fig. 4. Faraday's law relates the voltage induced in an electric crcuit that is coupled to a magnetic circuit.

$$v = N\frac{d\Phi}{dt} = \frac{N^2}{R}\frac{d\Phi}{dt}$$

The quantity N^2/R is defined as the inductance of the electric circuit.

$$v = L\frac{di}{dt}$$

Thus an electromagnetic circuit provides us an electric circuit element (inductor). The voltage across an inductor is directly proportional to the rate of rise of current through it. The energy stored in the magnetic circuit is

$$E = \frac{1}{2}L\,I^2 = \frac{1}{2}\frac{F^2}{R} = \frac{1}{2}\Phi^2 R = \frac{1}{2}\Phi F$$

The equivalent circuit of an inductor showing both its electric and magnetic parts may be conveniently represented as shown in Fig. 5. However in practice, the inductor will have certain parasitic resistance (of the wire in the electric circuit) and magnetic leakage (in the magnetic circuit). These non-idealities may be conveniently be incorporated in the equivalent circuit shown in Fig. 6. The design of an inductor involves the design of the electrical (Number of turns and wire size) and the magnetic (geometry of the magnetic core and its required magnetic property) circuit.

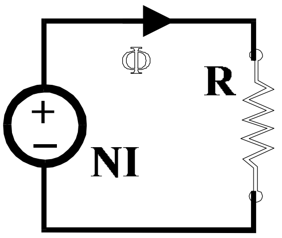


Fig. 4

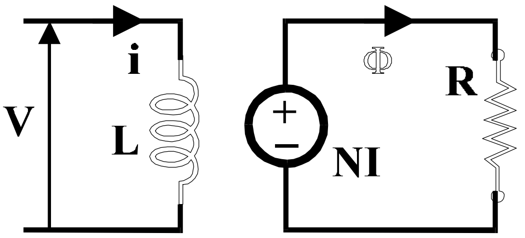


Fig. 5

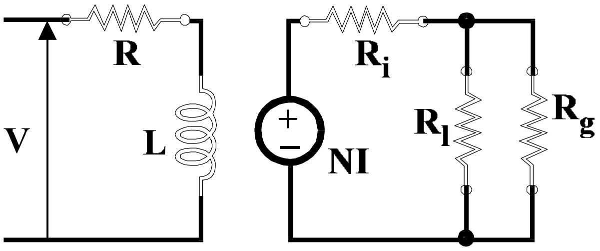


Fig. 6

DESIGN OF INDUCTOR

The inductor consists of a magnetic circuit and an electrical circuit. The design requires,

- ➢ The size of wire to be used for the electric circuit, to carry the rated current safely.
- ➢ The size and shape of magnetic core to be used such that
    - ➢ The peak flux is carried safely by the core without saturation.
    - ➢ The required size of the conductors are safely accommodated in the core.
- ➢ The number of turns of the electric circuit to obtain the desired inductance.

Material constraints

Any given wire (conducting material) can only carry a certain maximum current per unit cross section of the wire size. When this limit is exceeded, the wire will overheat from the heat generated (I^2R) and melt or deteriorate. The safe currentdensity for the conducting material is denoted by J A/m^2.

Any magnetic material can only carry a certain maximum flux density. When this limit is exceeded, the material saturates and the relative permeability μ_r drops substantially. This maximum allowable flux density for the magnetic material is denoted by B_m T.

Design Relationships

In order to design an inductor of L Henry, capable of carrying an rms current of I_{rms} and peak current of I_p,

- ☒ Let the wire size be a_w m^2

  $$a_w = \frac{I_{rms}}{J} \tag{1}$$

- ☒ Let the peak flux density in the core of area (A_c) be B_m T on account of the peak current I_p in the inductor.

  $$L\,I_p = N\,\Phi_p = N A_c B_m \tag{2}$$

- ☒ The winding of the inductor is accommodated in the window of the core. Let the window area (A_w) be filled by conductors to a fraction of k_w.

  $$k_w A_w = N\,a_w = N\,\frac{I_{rms}}{J} \tag{3}$$

  Cross multiplying Eq. (1) and Eq. (2), we get,

  $$L\,I_p\,N\,\frac{I_{rms}}{J} = N\,A_c\,B_m\,k_w\,a_w$$

  $$L\,I_p\,I_{rms} = k_w\,J\,B_m\,A_c\,A_w \tag{4}$$

- ☒ The above equation may be interpreted as a relationship between the energy handling capacity of the inductor to the size of the core (A_cA_w), the material properties (B_m,J), and our manufacturing skill (k_w).
- ➢ k_w depends on how well the winding can be accommodated in the window of the core. k_w is usually 0.3 to 0.5.

- Bm is the maximum unsaturated flux density for the core material. Bm is about 1 T for iron and 0.2 T for ferrites.
- J is the maximum allowable current density for the conductor. For copper conductors J is between $2 \cdot 10^6$ A/m^2 to $5 \cdot 10^6$ A/m^2

Design steps

Input: $L,\ Ip,\ Irms,$ Core Tables, Wire Tables, $J,\ Bm,\ kw.$

1 Compute $Ac\,Aw = \dfrac{L\,Ip\,Irms}{kw\,Bm\,J}$

2 Select a core from core tables with the required $Ac\,Aw$.

3 For the selected core, find $Ac,\ Aw$.

4 Compute $N = \dfrac{L\,Ip}{Bm\,Ac}$. Select nearest whole number of N^*.

5 Compute $aw = \dfrac{Irms}{J}$. Select next higher a_w^* from wire tables.

6 Compute the required air gap in the core $lg = \dfrac{\mu_o N^* Ip}{Bm}$.

7 Check for assumptions:

- Core reluctance << Airgap reluctance : $Rc << Rg$; $\dfrac{lc}{\mu r} << lg$
- No fringing : $lg << \sqrt{Ac}$

8 Recalculate $J^* = \dfrac{Irms}{a_w^*}$.

9 Recalculate $k_w^* = \dfrac{N^*\,a_w^*}{Aw}$.

10 Compute from the geometry of the core, mean length per turn and the length of the winding. From wire tables find the resistance of winding.

DESIGN OF TRANSFORMER

Unlike the inductor, the transformer consists of more than one winding. Also, in order to keep the magnetisation current low, the transformer does not have airgap in its magnetising circuit. Consider a transformer with a single primary and single secondary as shown in Fig. 7. Let the specifications be

Primary : V_1 volt ; I_1 ampere ;
Secondary : V_2 volt ; I_2 ampere ;
VA Rating : $V_1 I_1 = V_2 I_2$;
Frequency : f Hz.

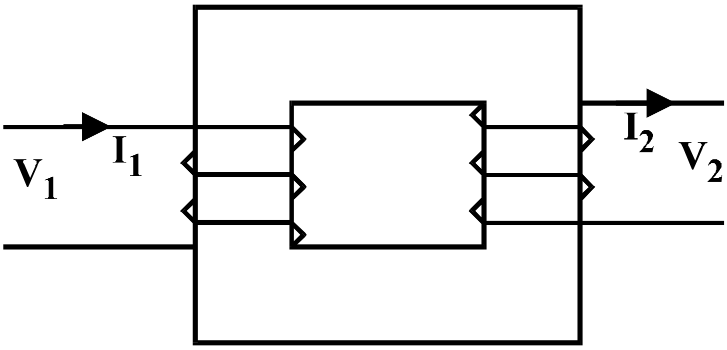


Fig. 7

Design Relationships

For square wave of operation, the voltage of the transformer is

$$V_1 = 4 f B_m A_c N_1 \quad ; \qquad V_2 = 4 f B_m A_c N_2$$

The window for the transformer accommodates both the primary and the secondary. With the same notation as for inductors,

$$k_w A_w = \frac{N_1 I_1 + N_2 I_2}{J} \quad ; \qquad N_1 I_1 = N_2 I_2 = \frac{k_w A_w J}{2}$$

From the above equations,

$$V_1 I + V_2 I_2 = 4 k_w J B_m f A_c A_w$$

$$VA = 2 k_w J B_m f A_c A_w$$

Design Steps

For a given specification of VA, V_1, V_2, J, B_m, k_w, and f it is desired to design a suitable transformer. The design requires

- Size of wire & number of turns to be used for primary and secondary windings.
- Core to be used.
- Resistance of the winding.
- Magnetising inductance of the transformer.

1. Compute the Area product ($A_c A_w$) of the desired core.
   $$A_c A_w = \frac{VA}{2 f k_w J B_m}$$
2. Select the smallest core from the core tables having an area product higher than obtained in step (1).
3. Find the core area (A_c) and window area (A_w) of the selected core.
4. Compute the number of turns. $N_1 = \dfrac{V_1}{4 f B_m A_c}$; $N_2 = \dfrac{V_2}{4 f B_m A_c}$
5. Select the nearest higher whole number to that obtained in step (4), for the primary & secondary turns.
6. Compute the wire size for secondary & primary. $a_{w1} = \dfrac{I_1}{J}$; $a_{w2} = \dfrac{I_2}{J}$
7. Select from the wire tables the desired wire size.
8. Compute the length of secondary & primary turns, from the mean length per turn of the core tables.
9. Find from the wire tables, the primary & secondary resistance.
10. Compute from the core details, the reluctance ($R = \dfrac{l_c}{A_c \mu_o \mu_r}$) of the core.
11. Compute the magnetising inductance. $L_m = \dfrac{N^2}{R}$

TRANSFORMER & CHOKE DESIGN TABLE

Laminations GKW Core Section :Square

Flux Density B_m 1 T for Inductor Current Density : J = 2.5 A/mm^2
1.2 T for Transformer

Window Space Factor k_w 0.3 Frequency (for Trafo) f = 50 Hz

Core Type Number	A_c mm^2	A_w mm^2	$A_p = A_c A$ mm^4	Maximum VA As a Trafo @	Maximum Energy As an Inductor #
L202	12.3	27.7	338.7	0.03	0.13
L164	23	53.3	1,227.6	0.12	0.46
L109	41	81.3	3,329	0.33	1.25
12AX	90.3	210.9	19,033	1.9	7.1
T 17	161.3	122.2	19,716	1.97	7.4
INT 41	169	168	28,392	2.8	10.6
17A	204.5	151.9	31,070	3.1	11.7
12A	252.8	188	47,533	4.7	17.8
10A	252.8	443.2	112,052	11.2	42
T 1	278.9	656.7	183,138	18.3	68.7
T 74	306.3	227.9	69,806	7	26.2
T 23	364.8	271.7	99,118	9.9	37.2
T 2	364.8	1,092.5	398,562	39.8	149.5
T 30	400	300	120,000	12	45
T45	492.8	369.6	182,168	18.2	68.3
T 31	492.8	369.6	182,168	18.2	68.3
T 15	645.2	483.9	312,173	31.2	117
T 14	645.2	656.7	423,657	42.3	159
T 33	784	588	460,992	46.1	173
T 3	1,011.2	756.8	765,346	76.5	287
T 16	1,451.6	1,092.5	1,585,913	158.4	595
T 5	1,451.6	1,269.8	1,843,312	184.1	691
T 6	1,451.6	1,935.5	2,809,562	280.7	1,054
INT 120	1,600	1,200	1,920,000	191.8	720
T 43	2,580.6	1,935.5	4,994,777	499	1,873
T 8	2,580.6	4,984.9	12,864,258	1,285	4,824
INT 180	3,600	2,700	9,720,000	971	3,645
8 A	5,806.4	7,096.8	41,206,911	4,117	15,452
8 B	5,806.4	4,984.9	28,944,581	2,892	10,854
8 C	5,806.4	9,965.7	57,865,181	5,781	21,699
T 100	10,322.6	11,612.9	119,874,000	11,975	44,953
4 AX	566.4	2,612.2	1,479,626	147.8	555
35 A	1,451.6	7,871.8	11,426,755	1,142	4,285.00
43 TP	645.2	2,903.2	1,873,041	281	
8 B TP	1,451.6	7,278.2	10,565,122	1,583	
100 TP	2,580.6	15,483.8	39,958,217	5,988	

Energy Capacity of the Inductor = $\frac{L I_{peak} I_{rms}}{2}$; @ Transformer Primary $V_{rms} I_{rms}$

Transformer Design: $N = \frac{3754V_{rms}}{A_c}$; $a_w\,(mm^2) = \frac{I_{rms}}{2.5}$;

Inductor Design: $N = \frac{10^6 L\, I_{peak}}{A_c}$; $a_w\,(mm^2) = \frac{I_{rms}}{2.5}$;

$l_g\,(mm) = 4\pi 10^{-4} N\, I_{peak}$

Nominal Diameter	WireSize SWG	Outer Diameter	Resistance Ohm/Km	Area in mm^2
0.025	50	0.036	34026	0.000506
0.030	49	0.041	23629	0.000729
0.041	48	0.051	13291	0.001297
0.051	47	0.064	8507	0.002027
0.061	46	0.074	5907	0.002919
0.071	45	0.086	4340	0.003973
0.081	44	0.097	3323	0.005189
0.091	43	0.109	2626	0.006567
0.102	42	0.119	2127	0.008107
0.112	41	0.132	1758	0.009810
0.122	40	0.142	1477	0.011675
0.132	39	0.152	1258	0.013701
0.152	38	0.175	945.2	0.018242
0.173	37	0.198	735.9	0.02343
0.193	36	0.218	589.1	0.02927
0.213	35	0.241	482.2	0.03575
0.234	34	0.264	402.0	0.04289
0.254	33	0.287	340.3	0.05067
0.274	32	0.307	291.7	0.05910
0.295	31	0.330	252.9	0.06818
0.315	30	0.351	221.3	0.07791
0.345	29	0.384	183.97	0.09372
0.376	28	0.417	155.34	0.1110
0.417	27	0.462	126.51	0.1363
0.457	26	0.505	105.02	0.1642
0.508	25	0.561	85.07	0.2027
0.559	24	0.612	70.30	0.2452
0.610	23	0.665	59.07	0.2919
0.711	22	0.770	43.40	0.3973
0.813	21	0.874	33.23	0.5189
0.914	20	0.978	26.26	0.6567
1.016	19	1.082	21.27	0.8107
1.219	18	1.293	14.768	1.167
1.422	17	1.501	10.850	1.589
1.626	16	1.709	8.307	2.075
1.829	15	1.920	6.564	2.627
2.032	14	2.129	5.317	3.243
2.337	13	2.441	4.020	4.289
2.642	12	2.756	3.146	5.480
2.946	11	3.068	2.529	6.818
3.251	10	3.383	2.077	8.302
3.658	9	3.800	1.640	10.51
4.064	8	4.219	1.329	12.97

CAPACITORS FOR POWER ELECTRONIC APPLICATIONS

Power electronic systems employ capacitors as power conditioning elements. Unlike in signal conditioning applications, the capacitors in PES are required to handle large power. As a result they must be capable of carrying large currents without overheating. To satisfy the demands in PES, the capacitors must be very close to their ideal characteristics - namely low equivalent series resistance (ESR) and low equivalent series inductance (ESL). Low ESR will ensure low losses in the capacitor. Low ESL will ensure that the capacitor can be used in a larger range of operating frequency. Figure 1 shows the impedance of a capacitor as a function of frequency. It is seen that a real capacitor is close to the ideal at lower frequencies. At higher frequencies, the ESR and the ESL of the real capacitor make it deviate from the ideal characteristics. For PES applications, it is necessary that the ESR and ESL of the capacitor are low.

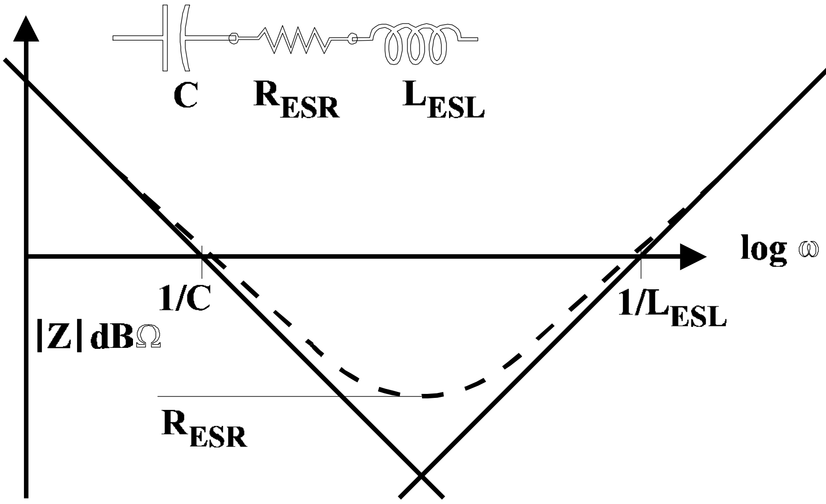


Fig. 1 Impedance of a Capacitor as a Function of Frequency

TYPES OF CAPACITORS

There are several different types of capacitors employed for power electronic applications.

1. Coupling capacitors

Coupling capacitors are used to transfer ac voltages between two circuits at different potentials. These occur at the interface between control circuits and power circuits in PES. The current carried by such a capacitor is comparatively low. The important feature of such capacitors is

- High insulation resistance.

A typical application is shown in Fig. 2.

2. Power capacitors (low frequency)

These are used in PES mainly to improve power factor. They are generally used at low frequencies (predominantly 50/60 Hz). They compensate the reactive power demanded by the load so that the power handling portion of the PES are not called upon to supply the reactive power. Further they also bypass harmonics generated in the PES. In such applications the voltage is predominantly sinusoidal; the current may be rich in harmonics. The important features of these capacitors are

- Capability to handle high reactive power.
- Capability to handle high harmonic current.

A typical application is shown in Fig. 3.

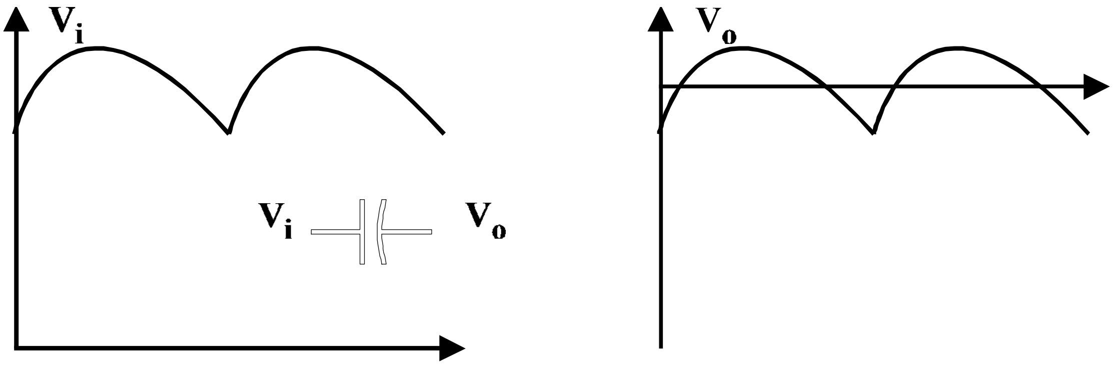


Fig. 2 Coupling Capacitor

3. Power capacitors (high frequency)

These are used for the same applications as the low frequency power capacitors but at higher frequencies (up to 20 KHz). Further they are also capable of carrying surge crrents resulting from switching. Such applications arise when capacitor banks are switched on and off to cater to conditions of varying load (typical in induction heating applications). The main features of these capacitors are

- Capability to handle large reactive power.
- Capability to operate at higher frequency.
- Capability to handle switching surge currents.

4. Filter capacitors

These capacitors are forward filtering capacitors to smooth out the variable source voltage applied to the load or reverse filtering capacitor to smooth out the variable load current from reaching the source. Theyare called upon to handle large periodic currents. The important features of these capacitors are

- High capacitance value.
- High rms current rating.

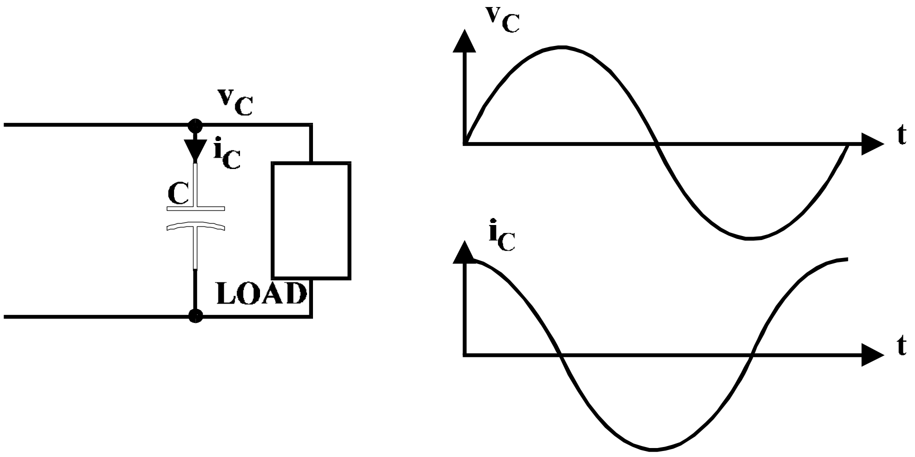


Fig. 3 Power Factor Improvement Capacitors

These capacitors are electrolytic capacitors on account of the unipolar voltage they are subjected to. Typical applicatios are shown in Fig. 4.

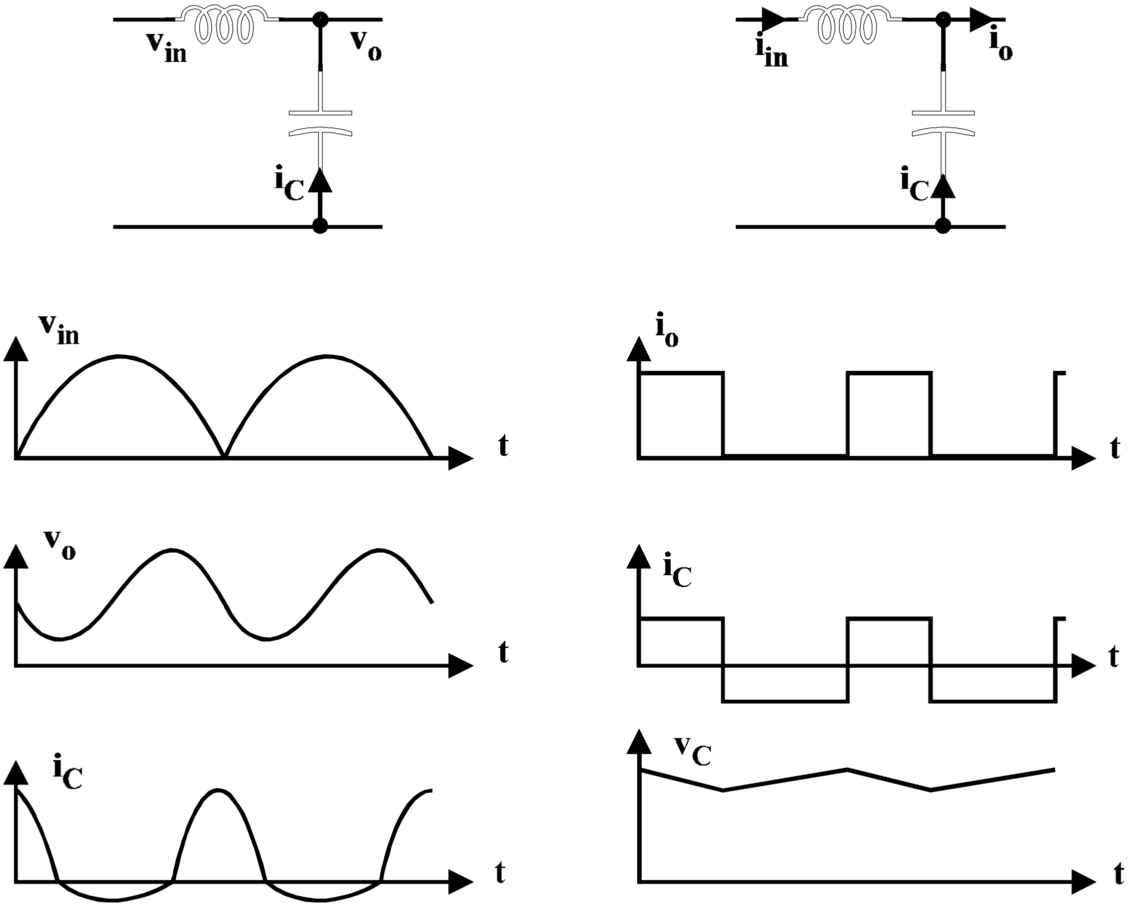


Fig. 4 Filter Capacitors

5. Pulse capacitors

Pulse capacitors are used to provide very high surge currents to loads. They will be charged over a relatively long period and discharged in a very short period. Typical applications are precision welding, electronic photo flash, electronic ignition etc. The rquired features for these applications are

- Large energy storage capacity.
- Large peak current handling capacity.
- Low ESL.

A typical application is shown in Fig. 5.

6. Damping capacitors

Damping or snubber capacitors are used in parallel with power switching devices to suppress undesired voltage stresses on the device. The rms currents in the capacitor will be high. The desired features are

- High rms current capacity.
- Low ESL.

A typical application is shown in Fig. 6.

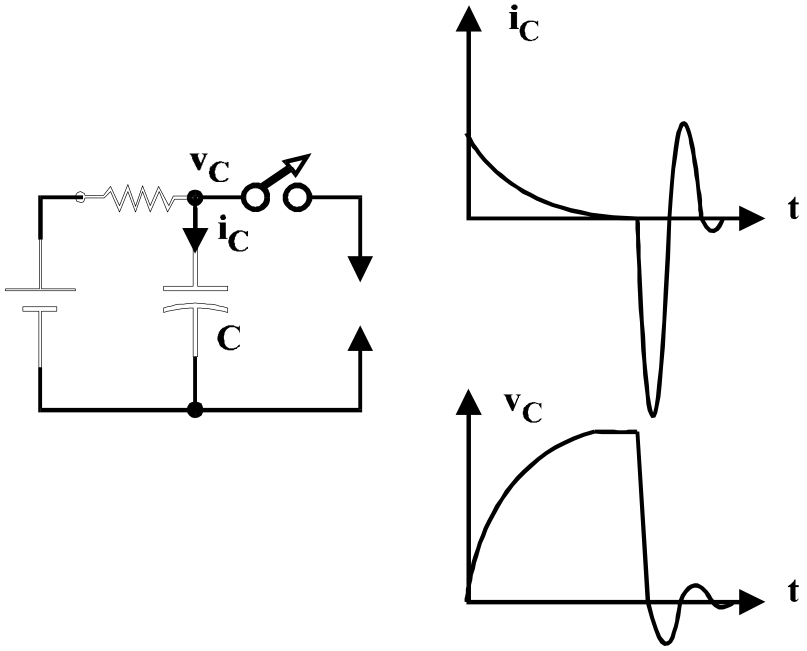


Fig. 5. Pulsed Energy Application

7. Commutation capacitors

These capacitors are employed in the commutation circuits of SCRs for forced turn-off of the device. They are subjected to very high reactive power and peak currents. The commutation process is quite short and so these capacitors must have purely capacitive reactances even at high operating frequency. The desired features are

- High peak current capacity.
- Low ESL.

A typical application is shown in Fig. 7.

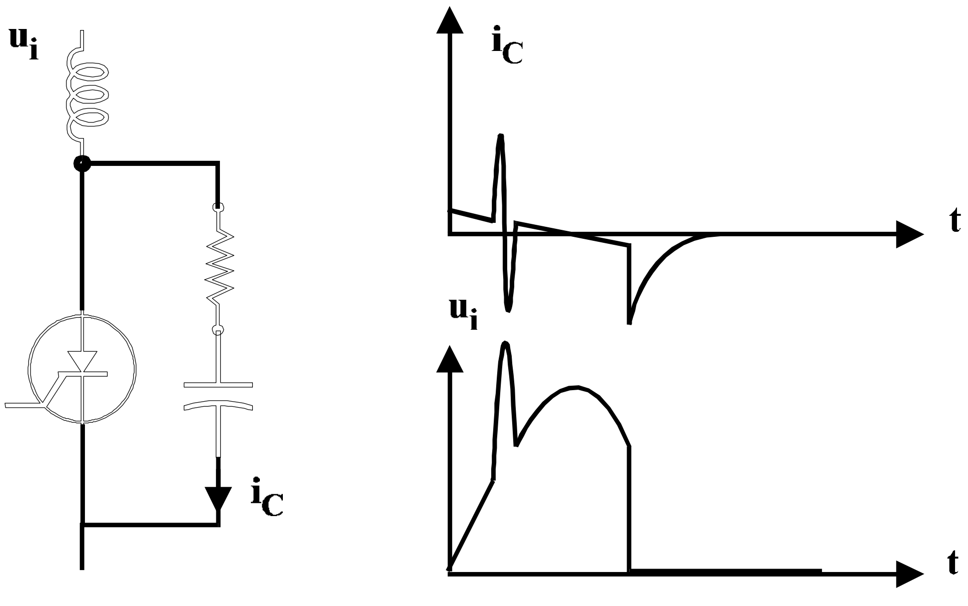


Fig. 6 Dampling Application

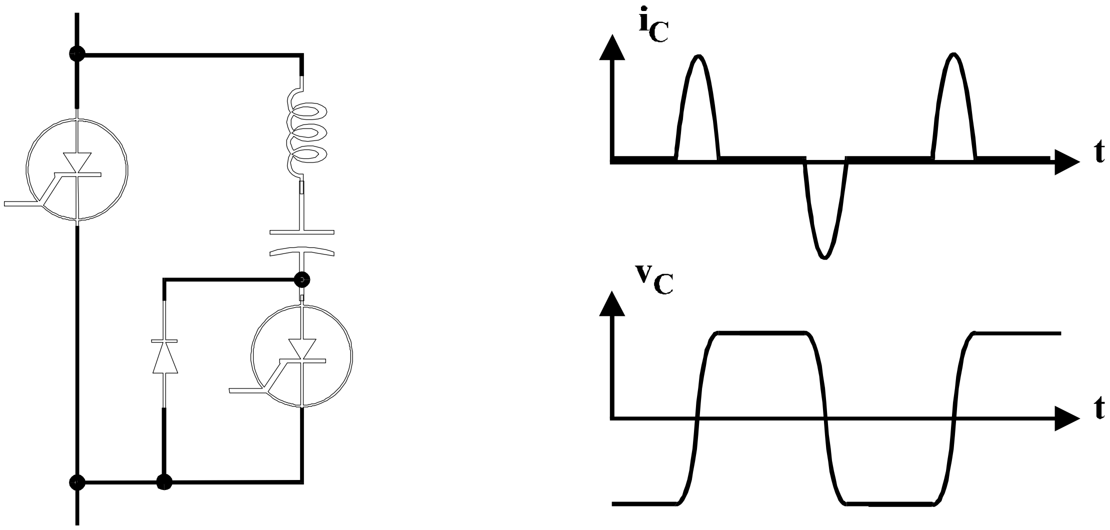


Fig. 7 A Commutation Application

8. Resonant capacitors

Resonant capacitors are used in circuits in combination with inductors and are subjected to sinusoidal voltages and currents. The operating frequency is high. The stability of the capacitor is important. The desirable features are

- Stability of capacitance.
- Low ESR

A typical application is shown in Fig. 8.

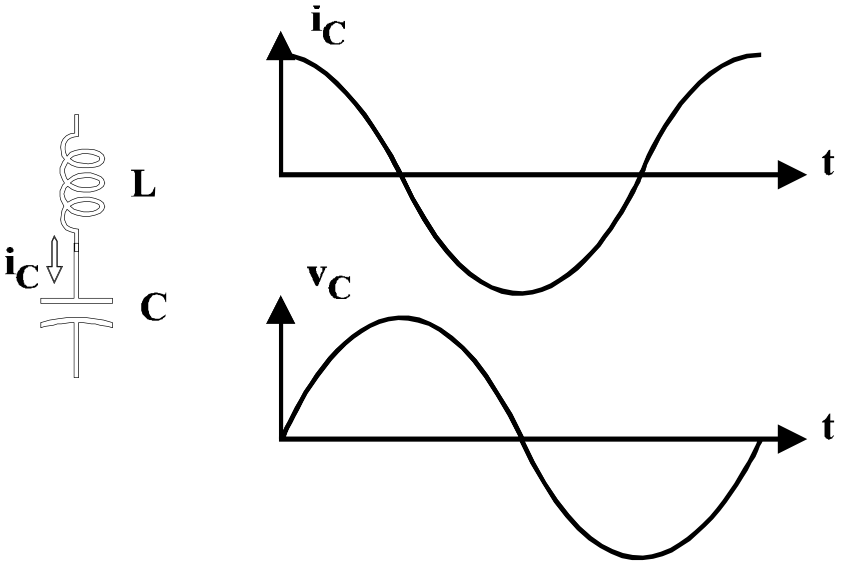


Fig. 8 Resonant Capacitor Application