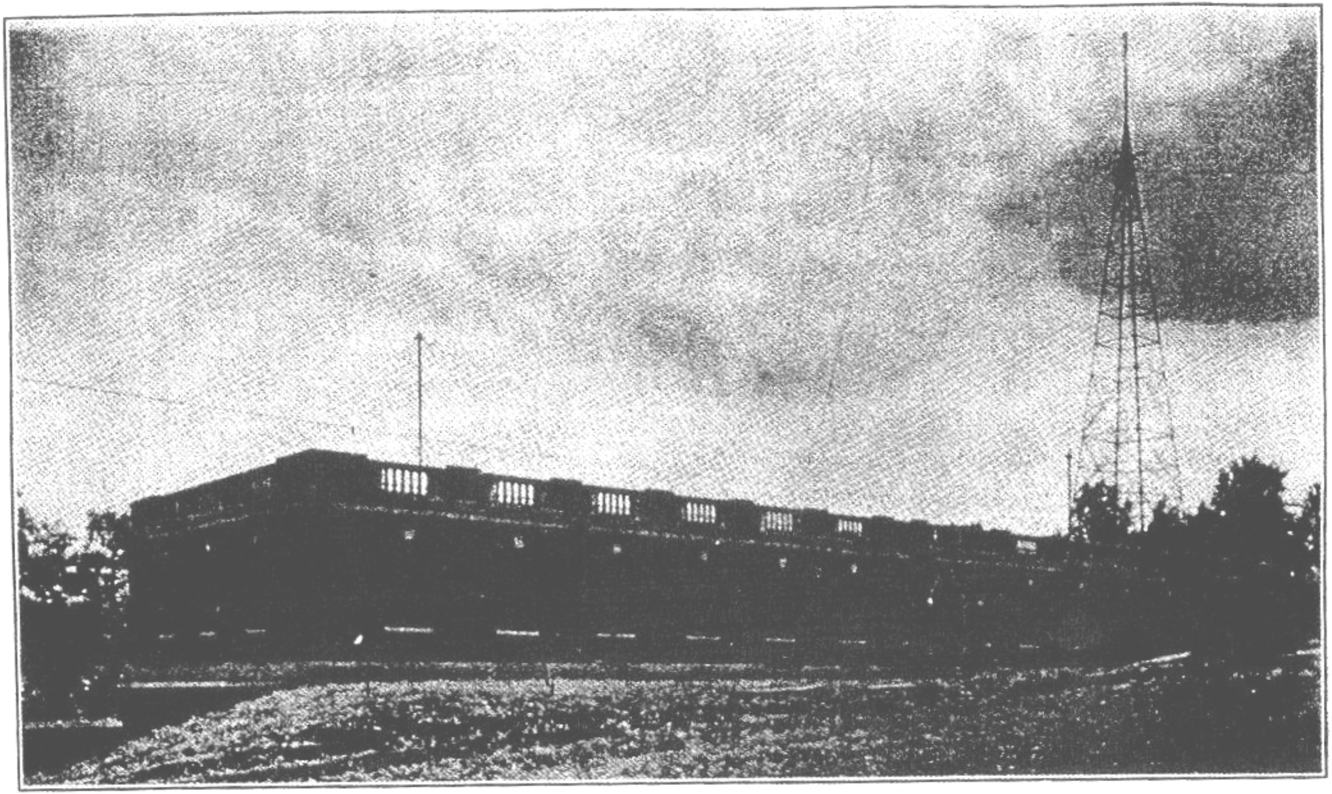
U. S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

RADIO INSTRUMENTS AND MEASUREMENTS

CIRCULAR C74



Radio laboratory of the Bureau of Standards

U. S. DEPARTMENT OF COMMERCE
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RADIO INSTRUMENTS AND MEASUREMENTS

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SYMBOLS USED IN THIS CIRCULAR

B = magnetic induction.
c = velocity of light = 2.9982×10^{10} cm per second.
C = electrostatic capacity.
d = diameter.
e = instantaneous electromotive force.
E = effective electromotive force.
E_o = maximum electromotive force.
$\mathscr{E}$ = electric field intensity.
f = frequency.
F = force.
$\mathscr{F}$ = magnetomotive force.
H = magnetic field intensity.
i = instantaneous current.
I = effective current.
I_o = maximum current.
$j = \sqrt{-1}$.
k = coupling coefficient.
K = dielectric constant.
l = length.
L = self-inductance.
m = mass.
M = mutual inductance.
p = instantaneous power.
P = average power.
Q = quantity of electricity.
r = distance from a point.
R = resistance.
s = length along a path.
S = area.
t = time.
T = period of a complete oscillation.
v = velocity.
V = potential difference of a condenser.
w = instantaneous energy.
W = average energy.
X = reactance.
Z = impedance.
δ = logarithmic decrement.
ϵ = base of napierian logarithms = 2.71828.
θ = phase angle.
λ = wave length.
μ = permeability.
ρ = volume resistivity.
ϕ = magnetic flux.
ψ = phase difference.
$\omega = 2\pi \times$ frequency.
μf = microfarad.
$\mu\mu f$ = micromicrofarad.
μh = microhenry.

Special symbols are defined where used in part III and elsewhere.

ERRATA

Page	
4	References to appendixes should be omitted; see symbols above and note below.
120	Par. 2, line next to last should end "micromicrofarads."
156	Fig. 111 (facing p. 156) legend should read "mounted thermoelement, etc."
160	Fig. 112, legend, last word should read "thermoelement."
174	Fig. 125, legend, last word should read "thermoelement."
193	Par. 1, after comma, read: "the resistance at constant frequency is approximately inversely proportional to the square of the setting, except near zero setting."
212	Par. 1 and legend to figure 148 are in error in stating that the coupling shown is electrostatic. The coupling is direct inductive.
226	Fig. 165, the direction of winding of S_1 should be reversed.
248	Eq 143, omit l after 0.002.
265	Eq 167, sign before 0.447 should be plus.
273	Eq 182, second plus sign in second line should be minus.
298	In first example omit 3d and 4th paragraphs. Number of turns is 25½.
300	Par. 2, line 2. Parameter should be multiplied by $\sqrt{\frac{1}{1000}}$.
300	Par. 6, eq 208. Constant should be 0.0107003.
301	Eq 210. Divide right-hand member by d.
311	Table 19, heading. Constant in equation should be 0.0107003.

NOTE.—This Circular is reprinted to meet a continuing demand. The errata above are referred to also in the margins at the proper points. Appendixes 1 and 2 are omitted.

CONTENTS

error
see p. 2

RADIO INSTRUMENTS AND MEASUREMENTS[a]

PART I—THEORETICAL BASIS OF RADIO MEASUREMENTS

INTRODUCTION

In the rapid growth of radio communication the appliances and methods used have undergone frequent and radical changes. In this growth progress has been made largely by new inventions and applications, and comparatively little attention paid to refinements of measurement. In consequence the methods and instruments of measurement peculiar to radio science have developed slowly and have not yet been carried to a point where they are as accurate or as well standardized as some other electrical measurements.

This circular presents information regarding the more important instruments and measurements actually used in radio work. The treatment is of interest to Government officers, radio engineers, and others. Many of the matters dealt with are or have been under investigation in the laboratories of the Bureau of Standards and are not treated in previously existing publications. No attempt is made in this circular to deal with the operation of apparatus in sending and receiving. The Bureau's publications on that and other radio subjects are listed in Appendix 2. The Bureau will appreciate suggestions from those who use this publication for improvements or changes which would make it more useful in future editions.

The methods, formulas, and data used in radio work can not be properly understood or effectively used without a knowledge of the principles on which they are based. The first part of this circular, therefore, attempts to give a summary of these principles

[a] The first edition of this circular, issued Mar. 23, 1918, was prepared by Messrs. J. H. Dellinger, J. M. Miller, and F. W. Grover, assisted by G. C. Southworth and other members of the radio laboratory. The revision for the second edition was done by Messrs. J. H. Dellinger, L. E. Whittemore, and R. S. Ould.

PART III.—FORMULAS AND DATA

CALCULATION OF CAPACITY

63. CAPACITY OF CONDENSERS

Units.—The capacities given by the following formulas are in micromicrofarads. This unit is 10^{-12} of the farad, the farad being defined as the capacity of a condenser charged to a potential of 1 volt by 1 coulomb of electricity. The micromicrofarad and the microfarad (one-millionth of a farad) are the units commonly used in radio work. Radio writers have occasionally used the cgs electrostatic unit, sometimes called the "centimeter." This unit is 1.1124 micromicrofarads.

In the formulas here given all lengths are expressed in centimeters and all areas in square centimeters. The constants given are correct [31] to 0.1 per cent.

PARALLEL PLATE CONDENSER

Let S = surface area of one side of one plate

τ = thickness of the dielectric

K = dielectric constant ($K = 1$ for air, and for most ordinary substances lies between 1 and 10).

$$C = 0.0885 K \frac{S}{\tau} \text{ micromicrofarads.} \tag{110}$$

If, instead of a single pair of metal plates, there are N similar plates with dielectric between, alternate plates being connected in parallel,

$$C = 0.0885 K \frac{(N-1)S}{\tau} \tag{111}$$

In these formulas no allowance is made for the curving of the lines of force at the edges of the plates; the effect is negligible when τ is very small compared with S.

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[31] The constants given in the formulas are correct for absolute units. To reduce to international units the values in absolute units should be multiplied by 1.00052. This difference need not be considered when calculations correct to 1 part in 1000 only are required.

VARIABLE CONDENSER WITH SEMICIRCULAR PLATES

Let N = total number of parallel plates
r_1 = outside radius of the plates
r_2 = inner radius of plates
τ = thickness of dielectric
K = dielectric constant

Then, for the position of maximum capacity (movable plates between the fixed plates),

$$C = 0.1390K\frac{(N-1)(r_1^2 - r_2^2)}{\tau} \tag{112}$$

This formula does not take into account the effect of the edges of the plates, but as the capacity is also affected by the containing case it will not generally be worth while to take the edge effect into account.

Formula (112) gives the maximum capacity between the plates with this form of condenser. As the movable plates are rotated the capacity decreases, and ordinarily the decrease in capacity is proportional to the angle through which the plates are rotated.

ISOLATED DISK OF NEGLIGIBLE THICKNESS

Let d = diameter of the disk
then

$$C = 0.354d \tag{113}$$

ISOLATED SPHERE

Let d = diameter of the sphere
then

$$C = 0.556\,d \tag{114}$$

TWO CONCENTRIC SPHERES

Let r_1 = inner radius of outside sphere
r_2 = radius of inside sphere
K = dielectric constant of material between the spheres

$$C = 1.112K\frac{r_1 r_2}{r_1 - r_2} \tag{115}$$

TWO COAXIAL CYLINDERS

Let r_1 = radius of outer cylinder
r_2 = radius of inner cylinder
K = dielectric constant of material between the cylinders
l = length of each cylinder

$$C = \frac{0.2416l}{\log_{10}\frac{r_1}{r_2}} \tag{116}$$

This formula makes no allowance for the difference in density of the charge as the ends of the cylinders are approached.

64. CAPACITY OF WIRES AND ANTENNAS.

SINGLE LONG WIRE PARALLEL TO THE GROUND

For a single wire of length l and diameter d, suspended at a height h above the ground, the capacity is

$$C = \frac{0.2416 l}{\log_{10} \frac{4h}{d} + \log_{10} \left\{ \frac{l/2 + \sqrt{l^2/4 + d^2/4}}{l/2 + \sqrt{l^2/4 + 4h^2}} \right\}} \qquad (117)$$

Usually the diameter d may be neglected in comparison with the length l, and the following equations are convenient for numerical computations.

For $\frac{4h}{l} \gtreqless 1$,

$$C = \frac{0.2416\, l}{\log_{10} \frac{4h}{d} - k_1} \qquad (118)$$

For $\frac{l}{4h} \gtreqless 1$,

$$C = \frac{0.2416\, l}{\log_{10} \frac{2l}{d} - k_2} \qquad (119)$$

in which the quantities

$$k_1 = \log_{10} \left\{ \frac{1 + \sqrt{1 + \left(\frac{4h}{l}\right)^2}}{2} \right\}$$

and

$$k_2 = \log_{10} \left\{ \frac{l}{4h} + \sqrt{1 + \left(\frac{l}{4h}\right)^2} \right\}$$

may be interpolated from Table 6, page 242.

These formulas assume a uniform distribution of charge from point to point of the wire.

VERTICAL WIRE

Formula (119), omitting the k_2 in the denominator, is sometimes used to calculate the capacity of a vertical wire. It applies accurately only when h is large compared with l, and gives very rough values for a vertical single-wire antenna, the lower end of which is connected to apparatus at least several meters above the ground.

CAPACITY BETWEEN TWO HORIZONTAL PARALLEL WIRES AT THE SAME HEIGHT

Let d = the diameter of cross section of the wires
l = length of each wire
h = the height of the wires above the earth
D = distance between centers of the wires.

The capacity is defined as the quotient of the charge on one wire, divided by the difference in potential of the two wires, when the potential of one wire is as much positive as the other is negative.

$$C=\frac{0.1208\,l}{\log_{10}\left\{\frac{l/2+\sqrt{l^2/4+d^2/4}}{l/2+\sqrt{l^2/4+4h^2}}\cdot\frac{4h}{d}\right\}-\log_{10}\left\{\frac{l/2+\sqrt{l^2/4+D^2}}{l/2+\sqrt{l^2/4+D^2+4h^2}}\cdot\frac{\sqrt{D^2+4h^2}}{D}\right\}}\qquad(120)$$

In most cases d/l and D/l may be neglected in comparison with unity, and we may write

$$C=\frac{0.1208\,l}{\log_{10}\frac{2D}{d}-\frac{D^2}{8h^2}}\qquad(121)$$

TWO PARALLEL WIRES, ONE ABOVE THE OTHER

For the case of one wire placed vertically above the other, the formula (121) may usually be used, taking for the value of h the mean height of the wires, $\frac{h_1+h_2}{2}$. The potential of one wire is assumed to be as much positive as the other is negative.

CAPACITY OF TWO PARALLEL WIRES JOINED TOGETHER

Let l = the length of each wire
D = distance between centers
h = their height above the earth
d = diameter of cross section.

The wires are supposed to be parallel to each other and to lie in a horizontal plane. They are joined together so that they are at the same potential. The capacity is defined as the quotient of the sum of their charges by the potential above the earth.

$$C=\frac{0.4831\,l}{\log_{10}\left\{\frac{l/2+\sqrt{l^2/4+d^2/4}}{l/2+\sqrt{l^2/4+4h^2}}\cdot\frac{4h}{d}\right\}+\log_{10}\left\{\frac{l/2+\sqrt{l^2/4+D^2}}{l/2+\sqrt{l^2/4+D^2+4h^2}}\cdot\frac{\sqrt{4h^2+D^2}}{D}\right\}}\qquad(122)$$

which, in those cases where d^2/l^2 and $\left(\frac{D}{2h}\right)^2$ may be neglected in comparison with unity, may be written in the following forms:

For $\frac{4h}{l} \gtrless 1$,

$$C = \frac{0.4831\, l}{\log_{10} \frac{4h}{d} + \log_{10} \frac{2h}{D} - 2k_1} \qquad (123)$$

For $\frac{l}{4h} \gtrless 1$,

$$C = \frac{0.4831\, l}{\log_{10} \frac{2l}{d} + \log_{10} \frac{l}{D} - 2k_2} \qquad (124)$$

The quantities k_1 and k_2 are the same as in (118) and (119) and may be obtained from Table 6, page 242.

These formulas assume a uniform distribution of charge along the wire.

CAPACITY OF A NUMBER OF HORIZONTAL WIRES IN PARALLEL

This case is of importance in the calculation of the capacity of certain forms of antenna. The wires are supposed to be joined together, and thus all are at the same potential. Their capacity in parallel is then defined as the quotient of the sum of all their charges by their common potential.

An expression for this case as accurate as the preceding formula (120) for two wires would be very complicated. The following simpler solution is nearly as accurate, and in view of the disturbing effect of trees, houses, and other like objects on the capacity of an antenna, will suffice for ordinary purposes of design.

Let n = number of wires in parallel
D = spacing of wires in parallel, measured between centers
d = diameter of wire
h = height of the wires above the ground
l = length of each wire.

Then if the potential coefficients be calculated as follows:

$$\left.\begin{aligned} p_{11} &= 4.605\left[\log_{10}\frac{4h}{d} - k_1\right] \\ p_{12} &= 4.605\left[\log_{10}\frac{2h}{D} - k_1\right] \end{aligned}\right\} \text{for } \frac{4h}{l} \gtrless 1, \qquad (125)$$

or,

$$\left.\begin{aligned} p_{11} &= 4.605\left[\log_{10}\frac{2l}{d} - k_2\right] \\ p_{12} &= 4.605\left[\log_{10}\frac{l}{D} - k_2\right] \end{aligned}\right\} \text{for } \frac{l}{4h} \gtrless 1, \qquad (126)$$

the approximate capacity of the n wires in parallel will be

$$C = 1.112l \div \left[\frac{p_{11} + (n-1)p_{12}}{n} - k\right] \tag{127}$$

the quantities k, k_1 and k_2 being obtained from Tables 6 and 7, page 242.

Example.—To find the capacity of an antenna of 10 wires 0.16 inch in diameter, in parallel, each wire 110 feet long, the spacing between the wires being 2 feet and their height above the ground 80 feet.

For this case $4h/l = \frac{320}{110}$ or $l/4h = 0.344$ and Table 6 gives $k_2 = 0.146$.

$$2l/d = \frac{2 \times 12 \times 110}{0.16} = 16500, \quad \log_{10}\frac{2l}{d} = 4.2175$$

$$l/D = \frac{110}{2} = 55 \qquad \log_{10} l/D = 1.7404$$

$$\therefore p_{11} = 4.605\,[4.218 - 0.146] = 18.75$$

$$p_{12} = 4.605\,[1.740 - 0.146] = 7.340$$

and from formula (127) and Table 7 the capacity is, reducing the length of the wires to cm

$$C = (1.112 \times 110 \times 30.5) \div \left[\frac{18.75 + 9(7.340)}{10} - 2.05\right]$$

$$= 584\ \mu\mu f = 0.000584\ \mu f.$$

Example.—A second antenna of 10 wires, 3/32 inch diameter, 155 feet long, spaced 2.5 feet apart, and stretched at a distance of 64 feet from the earth.

For this case $l/4h = \frac{155}{256} = 0.606, \qquad k_2 = 0.249$

$$2l/d = 39680, \quad \log_{10}\frac{2l}{d} = 4.5986$$

$$l/D = 62, \quad \log_{10} l/D = 1.7924$$

$$p_{11} = 20.04, \; p_{12} = 7.11, \; \frac{p_{11} + 9p_{12}}{10} - 2.05 = 6.35$$

$$C = \frac{1.112 \times 155 \times 30.5}{6.35} = 0.000829\ \mu f.$$

If the length of the antenna had been 500 feet, with the height unchanged, then $\frac{4h}{l}=\frac{256}{500}=0.512$, $k_1=0.026$, $\log_{10}\frac{4h}{d}=4.5154$, $\log_{10}\frac{2h}{D}=1.7093$; by (125) $p_{11}=20.67$, $p_{12}=7.75$, $k=2.05$,

$$C=\frac{1.112\times 500\times 30.5}{6.99}=0.002426\ \mu f.$$

65. TABLES FOR CAPACITY CALCULATIONS

TABLE 5.—For Converting Common Logarithms Into Natural Logarithms

Common	Natural	Common	Natural	Common	Natural	Common	Natural
0	0.0000	25.0	57.565	50.0	115.129	75.0	172.694
1.0	2.3026	26.0	59.867	51.0	117.432	76.0	174.996
2.0	4.6052	27.0	62.170	52.0	119.734	77.0	177.299
3.0	6.9078	28.0	64.472	53.0	122.037	78.0	179.602
4.0	9.2103	29.0	66.775	54.0	124.340	79.0	181.904
5.0	11.513	30.0	69.078	55.0	126.642	80.0	184.207
6.0	13.816	31.0	71.380	56.0	128.945	81.0	186.509
7.0	16.118	32.0	73.683	57.0	131.247	82.0	188.812
8.0	18.421	33.0	75.985	58.0	133.550	83.0	191.115
9.0	20.723	34.0	78.288	59.0	135.853	84.0	193.417
10.0	23.026	35.0	80.590	60.0	138.155	85.0	195.720
11.0	25.328	36.0	82.893	61.0	140.458	86.0	198.022
12.0	27.631	37.0	85.196	62.0	142.760	87.0	200.325
13.0	29.934	38.0	87.498	63.0	145.063	88.0	202.627
14.0	32.236	39.0	89.801	64.0	147.365	89.0	204.930
15.0	34.539	40.0	92.103	65.0	149.668	90.0	207.233
16.0	36.841	41.0	94.406	66.0	151.971	91.0	209.535
17.0	39.144	42.0	96.709	67.0	154.273	92.0	211.838
18.0	41.447	43.0	99.011	68.0	156.576	93.0	214.140
19.0	43.749	44.0	101.314	69.0	158.878	94.0	216.443
20.0	46.052	45.0	103.616	70.0	161.181	95.0	218.746
21.0	48.354	46.0	105.919	71.0	163.484	96.0	221.048
22.0	50.657	47.0	108.221	72.0	165.786	97.0	223.351
23.0	52.959	48.0	110.524	73.0	168.089	98.0	225.653
24.0	55.262	49.0	112.827	74.0	170.391	99.0	227.956
						100.0	230.259

The table is carried out to a higher precision than the formulas, e. g., 2.3026 is abbreviated to 2.303 in the formulas.

Examples.—To illustrate the use of such a table, suppose we wish to find the natural logarithm of 37.48. The common logarithm of 37.48 is 1.57380.

If we denote the number 2.3026 by M, then from the table

$$\begin{array}{ll} 1.5 & M=3.4539 \\ .073 & M=\ .1681 \\ .00080 & M=\ .0018 \\ \hline & 3.6238=\log_e 37.48 \end{array}$$

To find the natural logarithm of 0.00748: The common logarithm is $\bar{3}.87390$, which may be written $0.87390-3$. Entering the table we find

$$\begin{array}{lll} 0.87 & M=2.00325 & -3\,M=-6.9078 \\ .0039 & M=\ .00898 & \\ \hline \text{sum} & 2.0122 & \\ & -6.9078 & \\ \hline & -4.8956 & =\text{natural log of } 0.00748 \end{array}$$

TABLE 6.—For Use in Connection with Formulas (118), (119), (123), (124), (125), and (126)

4h/l	k_1	l/4h	k_2	4h/l	k_1	l/4h	k_2
0	0	0	0	0.6	0.035	0.6	0.247
0.1	0.001	0.1	0.043	.7	.045	.7	.283
.2	.004	.2	.086	.8	.057	.8	.318
.3	.009	.3	.128	.9	.069	.9	.351
.4	.016	.4	.169	1.0	.082	1.0	.383
.5	.025	.5	.209				

TABLE 7.—Values of k in Formulas (127) and (146)

n	k	n	k	n	k	n	k
2	0	6	1.18	11	2.22	16	2.85
3	0.308	7	1.43	12	2.37	17	2.95
4	.621	8	1.66	13	2.51	18	3.04
5	.906	9	1.86	14	2.63	19	3.14
		10	2.05	15	2.74	20	3.24

CALCULATION OF INDUCTANCE

66. GENERAL

In this section are given formulas for the calculation of self and mutual inductance in the more common circuits met with in practice. The attempt is here made, not to present all the formulas available for this purpose, but rather the minimum number required, and to attain an accuracy of about one part in a thousand. So far as has seemed practicable, tables have been prepared to facilitate numerical calculations. In some cases, to render interpolation more certain, the values in the tables are carried out to one more significant figure than is necessary. In such instances, after having obtained the required quantity by interpolation from a table, the superfluous figure may be dropped. In all the tables the intervals for which the desired quantities are tabulated are taken small enough to render the consideration of second differences in interpolation unnecessary.

Most of the formulas given are for low frequencies, this fact being indicated by the subscript zero, thus L_o, M_o. The high-frequency formulas are given where such are known. Fortunately it is possible by proper design to render unimportant the change of inductance with frequency, except in cases where extremely high precision is required.

The usual unit of inductance used in radio work is the microhenry, which is one millionth of the international henry.[32] The

[32] The constants in the formulas for inductance given here refer to absolute units. To reduce to international units multiply by 0.99948. Since, however, an accuracy of the order of only one part in a thousand is sought here, it will not be necessary to take this difference into account.

henry is defined as the inductance "in a circuit when the electromotive force induced in this circuit is one international volt, while the inducing current varies at the rate of one ampere per second." 1 henry = 1000 millihenries = 10^6 microhenries = 10^9 *cgs* electromagnetic units.

In the following formulas lengths and other dimensions are expressed in centimeters, unless otherwise stipulated, and the inductance calculated will be in microhenries.

Logarithms are given, either to the natural base ϵ or to the base 10, as indicated. The labor involved in the multiplication of common logarithms by the factor 2.303 to reduce to the corresponding natural logarithms will be very materially reduced by the employment of the multiplication table, Table 5, page 241, which is an abridgement of the table for this purpose usually given in collections of logarithms.

All of these formulas assume that there is no iron in the vicinity of the conductor or circuit of which the inductance is to be calculated. Thus, the formulas here given can not be used to calculate the inductance of electromagnets.

A much more complete collection of inductance formulas with numerical examples is given in the Bulletin of the Bureau of Standards, **8**, pages 1–237; 1912; also known as Scientific Paper No. 169.

67. SELF-INDUCTANCE OF WIRES AND ANTENNAS

STRAIGHT, ROUND WIRE

If l = length of wire
d = diameter of cross section
μ = permeability of the material of the wire

$$L_0 = 0.002l \left[\log_\epsilon \frac{4l}{d} - 1 + \frac{\mu}{4} \right] \text{ microhenries} \tag{128}$$

$$= 0.002l \left[2.303 \log_{10} \frac{4l}{d} - 1 + \frac{\mu}{4} \right] \text{ microhenries} \tag{129}$$

For all except iron wires this becomes

$$L_0 = 0.002l \left[2.303 \log_{10} \frac{4l}{d} - 0.75 \right] \tag{130}$$

For wires whose length is less than about 1000 times the diameter of the cross section $\left(\frac{2l}{d} < 1000\right)$, the term $\frac{d}{2l}$ should be added inside the brackets. These formulas give merely the self-inductance of one conductor. If the return conductor is not far away, the mutual inductances have to be taken into account (see formulas (134) and (136)).

As the frequency of the current increases, the inductance diminishes, and approaches the limiting value

$$L_{\infty}=0.002l\left[2.303 \log_{10}\frac{4l}{d}-1\right] \tag{131}$$

which holds for infinite frequency.

The general formula for the inductance at any frequency is

$$L=0.002l\left[2.303 \log_{10}\frac{4l}{d}-1+\mu\delta\right] \tag{132}$$

where δ is a quantity given in Table 8, page 282, as a function of x where

$$x=0.1405d\sqrt{\frac{\mu f}{\rho}} \tag{133}$$

f = frequency.

ρ = volume resistivity of wire in microhm-centimeters

ρ_c = same for copper

$\mu=1$ for all except iron wires.

For copper at 20° C, $x_c=0.1071\ d\sqrt{f}$.

The value a_c of x for a copper wire 0.1 cm in diameter at different frequencies may be obtained from Table 19, page 311. For a copper wire d cm in diameter $x_c=10\ d\ a_c$ and for a wire of some other material $x=10\ d\ a_c\sqrt{\mu\frac{\rho_c}{\rho}}$.

The total change in inductance when the frequency of the current is raised from zero to infinity is a function of the ratio of the length of the wire to the diameter of the cross section. Thus, the decrease in inductance of a wire whose length is 25 times the diameter is 6 per cent at infinite frequency; and for a wire 100 000 times as long as its diameter, 2 per cent.

Example.—For a copper wire of length 206.25 cm and diameter 0.25 cm at a wave length of 600 meters, that is $f=500\ 000$, the value of x is 18.93, and from Table 8, $\delta=0.037$.

$$\mu=1,\quad \frac{4l}{d}=3300,\quad \log_{10} 3300=3.51851$$

(From Table 5)

$$\begin{array}{r} \log_e 3300 = 8.0590 \\ 414 \\ 12 \\ \hline 8.1016 \end{array}$$

For zero frequency

$$L_0 = 0.4\,[8.102 - 1 + 0.25] = 2.941 \text{ microhenry}$$

For $f = 500\,000$

$$L = 0.4\,[8.102 - 1 + 0.037] = 2.856 \text{ microhenry}$$

a difference of 2.9 per cent out of a possible 3.4 per cent.

For an iron wire of the same length and diameter, assuming a resistivity 7 times as great as that of copper, and a permeability of 100, the value of x is $\sqrt{\frac{100}{7}}$ times as great as for the copper wire, or 71.5, and for this value of x,

$$\delta = 0.010 \text{ (Table 8)}$$

$$L_0 = 0.4\,[32.10] = 12.84\ \mu h$$

$$L = 0.4\,[8.102] = 3.24\ \mu h \text{ at } 500\,000 \text{ cycles.}$$

The limiting value is $L_\infty = 2.84\ \mu h$.

TWO PARALLEL, ROUND WIRES—RETURN CIRCUIT

In this case the current is supposed to flow in opposite directions in two parallel wires each of length l and diameter d. Denoting by D the distance from the center of one wire to the center of the other,

$$L = 0.004\,l\left[2.303 \log_{10} \frac{2D}{d} - \frac{D}{l} + \mu\delta\right] \qquad (134)$$

The permeability of the wires being μ, and δ being obtained from (133) and Table 8, page 282. For low frequency $\delta = 0.25$. This formula neglects the inductance of the connecting wires between the two main wires. If these are not of negligible length, their inductances may be calculated by (132) and added to the result obtained by (134), or else the whole circuit may be treated by the formula (138) for the rectangle below.

STRAIGHT RECTANGULAR BAR

Let l = length of bar.

b, c = sides of the rectangular section.

$$L_0 = 0.002\, l \left[2.303 \log_{10} \frac{2l}{b+c} + 0.5 + 0.2235 \frac{(b+c)}{l} \right] \quad (135)$$

The last term may be neglected for values of l greater than about 50 times $(b+c)$.

The permeability of the wire is here assumed as unity.

RETURN CIRCUIT OF RECTANGULAR WIRES

If the wires are supposed to be of the same cross section, b by c, and length l, and of permeability unity, and the distance between their centers is D,

$$L_0 = 0.004\, l \left[2.303 \log_{10} \frac{D}{b+c} + \frac{3}{2} - \frac{D}{l} + 0.2235 \frac{(b+c)}{l} \right] \quad (136)$$

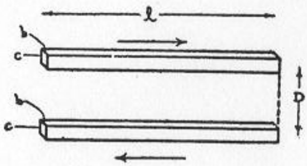

FIG. 178.—*The two conductors of a return circuit of rectangular wires*

For wires of different sizes, the inductance is given by $L_0 = L_1 + L_2 - 2M$ in which the inductances L_1 and L_2 of the individual wires are to be calculated by (135), and their mutual inductance M by (174) below.

SQUARE OF ROUND WIRE

If a is the length of one side of the square and the wire is of circular cross section of diameter d, the permeability of the wire being μ,

$$L = 0.008\, a \left[2.303 \log_{10} \frac{2a}{d} + \frac{d}{2a} - 0.774 + \mu\delta \right] \quad (137)$$

in which δ may be obtained from Table 8 as a function of the argument x given in formula (133). The value of δ for low frequency is 0.25, and for infinite frequency is 0.

RECTANGLE OF ROUND WIRE

Let the sides of the rectangle be a and a_1, the diagonal $g=\sqrt{a^2+a_1^2}$ and d = diameter of the cross section of the wire. Then the inductance at any frequency is

$$L=0.00921\left[(a+a_1)\log_{10}\frac{4aa_1}{d}-a\log_{10}(a+g)-a_1\log_{10}(a_1+g)\right]$$
$$+0.004\,[\mu\delta\,(a+a_1)+2\,(g+d/2)-2\,(a+a_1)] \qquad (138)$$

The quantity δ is obtained by use of (133) and Table 8. Its value for zero frequency is 0.25, and is 0 for infinite frequency.

RECTANGLE OF RECTANGULAR-SECTION WIRE

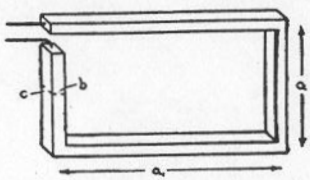

FIG. 179.—*Rectangle of rectangular wire*

Assuming the dimensions of the section of the wire to be b and c, and the sides of the rectangle a and a_1, then for nonmagnetic material the inductance at low frequency is

$$L_0=0.00921\left[(a+a_1)\log_{10}\frac{2aa_1}{b+c}-a\log_{10}(a+g)-a_1\log_{10}(a_1+g)\right]$$
$$+0.004\left[2g-\frac{a+a_1}{2}+0.447\,(b+c)\right] \qquad (139)$$

where $g=\sqrt{a^2+a_1^2}$.

INDUCTANCE OF GROUNDED HORIZONTAL WIRE

If we have a wire placed horizontally with the earth, which acts as the return for the current, the self-inductance of the wire is given by the following formula, in which

l = length of the wire
h = height above ground
d = diameter of the wire
μ = permeability of the wire
δ = constant given in Table 8, to take account of the effect of frequency (see p. 282).

$$L=0.004605\,l\left[\log_{10}\frac{4h}{d}+\log_{10}\left\{\frac{l+\sqrt{l^2+d^2/4}}{l+\sqrt{l^2+4h^2}}\right\}\right]$$
$$+0.002\left[\sqrt{l^2+4h^2}-\sqrt{l^2+d^2/4}+\mu l\delta-2h+\frac{d}{2}\right] \qquad (140)$$

which, neglecting $\frac{d}{l}$, as may be done in all practical cases, may be written in the following forms convenient for calculation:

For $\frac{2h}{l} \leqq 1$,

$$L = 0.002\, l \left[2.3026 \log_{10} \frac{4h}{d} - P + \mu\delta \right] \qquad (141)$$

and for $\frac{l}{2h} \leqq 1$,

$$L = 0.002\, l \left[2.3026 \log_{10} \frac{4l}{d} - Q + \mu\delta \right] \qquad (142)$$

the values of P and Q being obtained by interpolation from Table 9, page 283.

Mutual Inductance of Two Parallel Grounded Wires.—The two wires are assumed to be stretched horizontally, with both ends grounded, the earth forming the return circuit.

Let l = length of each wire
 d = diameter of wire
 D = distance between centers of the wires
 h = height above the earth

Then

$$M = 0.004605\, l \left[\log_{10} \frac{\sqrt{4h^2 + D^2}}{D} + \log_{10} \left\{ \frac{l + \sqrt{l^2 + D^2}}{l + \sqrt{l^2 + D^2 + 4h^2}} \right\} \right]$$

error see p. 2

$$+ 0.002\, l \left[\sqrt{l^2 + D^2 + 4h^2} - \sqrt{l^2 + D^2} + D - \sqrt{D^2 + 4h^2} \right] \qquad (143)$$

which, if we neglect $\frac{D^2}{l^2}$ and $\left(\frac{D}{2h}\right)^2$ may be expressed in the following forms:

For $\frac{2h}{l} \leqq 1$,

$$M = 0.002\, l \left[2.3026 \log_{10} \frac{2h}{D} - P + \frac{D}{l} \right] \qquad (144)$$

and for $\frac{l}{2h} \leqq 1$,

$$M = 0.002\, l \left[2.3026 \log_{10} \frac{2l}{D} - Q + \frac{D}{l} \right] \qquad (145)$$

the values of the quantities P and Q being obtained by interpolation from Table 9.

INDUCTANCE OF GROUNDED WIRES IN PARALLEL

The expressions for the inductance of n grounded wires in parallel involve the inductances of the single wires and the mutual inductances between the wires. Even in the case that the wires are all alike and evenly spaced, these expressions are very complicated.

The following approximate equation, which neglects the resistances of wires, is capable of giving results accurate to perhaps 1 per cent, for n wires of the same diameter evenly spaced.

Calculate by equations (141), (142), (144), or (145) the inductance L_1 per unit length of a single wire and the mutual inductance M_1 per unit length of any two adjacent wires using, of course, the actual length in the calculation of the ratios $\frac{2h}{l}$, $\frac{2l}{d}$, etc. Then

$$L = l\left[\frac{L_1+(n-1)\ M_1}{n} - 0.001\ k\right] \qquad (146)$$

in which n is the number of wires in parallel and k is a function of n tabulated in Table 7, page 242.

Example.—An antenna of 10 wires in parallel, each wire 155 feet long and $\frac{3}{32}$ inch in diameter, spaced 2.5 feet apart, and suspended at a height of 64 feet above the earth. Find the inductance at 100 000 cycles per second.

We have here $\frac{2h}{l} = \frac{128}{155} = 0.826$, and using this as argument in Table 9, $P = 0.6671$.

From (133) $x = 8.07$, and thence from Table 8, $\delta = 0.087$.

$$\frac{4h}{d} = 256 \times 12 \times \frac{32}{3} = 32\ 768,\quad \log_{10}\frac{4h}{d} = 4.515$$

$$\frac{2h}{D} = \frac{128}{2.5} = 51.2 \qquad\qquad \log_{10}\frac{2h}{D} = 1.709$$

Then, from formulas (141) and (144)

$$L_1 = 0.002[4.515 \times 2.3026 - 0.667 + 0.087]$$
$$= 0.01963\ \mu h \text{ per cm}$$

$$M_1 = 0.002[1.709 \times 2.3026 - 0.667 + 0.016]$$
$$= 0.006568\ \mu h \text{ per cm.}$$

From Table 7 we find for $n=10$, $k=2.05$, so that the inductance as calculated by (146) is

$$L=155\times 30.5\left[\frac{0.01963+9(0.006568)}{10}-0.00205\right]$$
$$=4727\,[0.00582]=27.4\ \mu h.$$

CIRCULAR RING OF CIRCULAR SECTION

If a = mean radius of ring

d = diameter of wire, the inductance at any frequency is, except for values of $\frac{d}{2a}>0.2$,

$$L=0.01257\,a\left\{2.303\log_{10}\frac{16a}{d}-2+\mu\delta\right\} \tag{147}$$

in which δ will be obtained from (133) and Table 8, page 282. Its value for zero frequency is 0.25.

TUBE BENT INTO A CIRCLE

Let the inner and outer diameters of the annular cross section of the tube be d_1 and d_2, respectively, and the mean radius of the circle a, then neglecting $\frac{d_1^2}{a^2}$ and $\frac{d_2^2}{a^2}$

$$L_0=0.01257\,a\left[2.303\log_{10}\frac{16a}{d_2}-1.75-\frac{d_1^2}{2(d_2^2-d_1^2)}+2.303\frac{d_1^4}{(d_2^2-d_1^2)^2}\log_{10}\frac{d_2}{d_1}\right] \tag{148}$$

For infinite frequency this becomes

$$L_\infty=0.01257\,a\left[2.303\log_{10}\frac{16a}{d_2}-2\right] \tag{149}$$

68. SELF-INDUCTANCE OF COILS

CIRCULAR COIL OF CIRCULAR CROSS SECTION

For a coil of n fine wires wound with the mean radius of the turns equal to a, the area of cross section of the winding being a circle of diameter d,

$$L_0=0.01257\,an^2\left\{2.303\log_{10}\frac{16a}{d}-1.75\right\} \tag{150}$$

This neglects the space occupied by the insulation between the wires.

TORUS WITH SINGLE-LAYER WINDING

A torus is a ring of circular cross section (doughnut shape).

Let R = distance from axis to center of cross section of the winding
a = radius of the turns of the winding
n = number of turns of the winding

$$L_o = 0.01257\, n^2 \left[R - \sqrt{R^2 - a^2}\right] \quad (151)$$

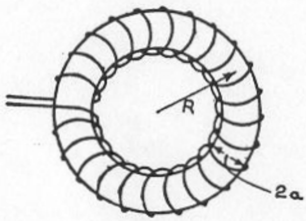

FIG. 180.—*Torus of single layer winding*

TOROIDAL COIL OF RECTANGULAR CROSS SECTION WITH SINGLE-LAYER WINDING

A coil of this shape might also be called a circular solenoid of rectangular section.

Let r_1 = inner radius of toroid (distance from the axis to inside of winding)
r_2 = outer radius of toroid (distance from axis to outside of winding)
h = axial depth of toroid.

Then $L_o = 0.004606\, n^2\, h \log_{10} \frac{r_2}{r_1}$ (152)

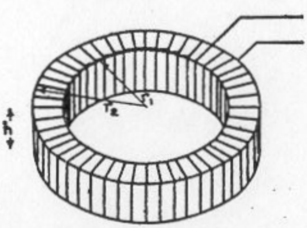

FIG. 181.—*Toroidal coil of rectangular section with single layer winding*

The value so computed is strictly correct only for an infinitely thin winding. For a winding of actual wires a correction may be calculated as shown in Bulletin, Bureau of Standards, 8, page 125; 1912. The correction is, however, very small.

SINGLE-LAYER COIL OR SOLENOID

An approximate value is given by

$$L_s = \frac{0.03948\ a^2 n^2}{b} K \tag{153}$$

where n = number of turns of the winding, a = radius of the coil, measured from the axis to the center of any wire, b = length of coil = n times the distance between centers of turns, and K is a function of $\frac{2a}{b}$ and is given in Table 10, page 283, which was calculated by Nagaoka. (See Bulletin, Bureau of Standards, 8, p. 224, 1912.) For a coil very long in comparison with its diameter, $K = 1$.

Formula (153) takes no account of the shape or size of the cross section of the wire. Formulas are given below for more accurate calculation of the low-frequency inductance. The inductance at high frequency can not generally be calculated with great accuracy. Formulas which take account of the skin effect, or change of current distribution with frequency, have been developed. The change is very small when the coil is wound with suitably stranded wire. The inductance at high frequencies depends, however, also on the capacity of the coil, which is generally not calculable. If the capacity is known, from measurements or otherwise, its effect upon the inductance can be calculated by

$$L_a = L\ [1 + \omega^2\ CL(10)^{-18}] \tag{154}$$

where L_a is the apparent or observed value of the inductance, C is in micromicrofarads, and L in microhenries. The inductance of a coil is decreased by skin effect, and is increased by capacity. The changes due to these two effects sometimes neutralize each other, and in general, formula (153) gives about as good a value of the high-frequency inductance as can be obtained.

Round Wire.—The low-frequency inductance of a coil wound with round wire can be calculated to much higher precision than that of formula (153) by the use of correction terms. Formula (153) gives strictly, the inductance of the equivalent current sheet, which is a winding in which the wire is replaced by an extremely thin tape, the center of each turn of tape being situated at the center of a turn of wire, the edges of adjacent tapes being separated by an infinitely thin insulation. The inductance of the actual coil is obtained from the current-sheet inductance as follows:

Putting L_s = inductance of equivalent cylindrical current sheet, obtained from (153)

L_0 = inductance of the coil at low frequencies

n = number of turns

a = radius of coil measured out to the center of the wire

D = pitch of winding = distance from center of one wire to the center of the next measured along the axis

b = length of equivalent current sheet $= nD$

d = diameter of the bare wire

Then $\quad L_0 = L_s - 0.01257\ na\ (A+B)$ microhenry $\quad$ (155)

in which A is constant, which takes into account the difference in self-inductance of a turn of the wire from that of a turn of the current sheet, and B depends on the difference in mutual inductance of the turns of the coil from that of the turns of the current sheet. The quantities A and B may be interpolated from Tables 11 and 12, page 284, which are taken from Tables 7 and 8 of Bulletin, Bureau of Standards, 8, pages 197–199; 1912. (Sci. Paper 169.)

Example.—A coil of 400 turns of round wire of bare diameter 0.05 cm, wound with a pitch of 10 turns per cm, on a form of such a diameter that the mean radius out to the center of the wire is 10 cm.

$$a = 10,\ \ b = nD = 40,\ \ n = 400,\ \ D = 0.1,\ \frac{d}{D} = 0.5$$

The value of K corresponding to $\frac{2a}{b} = 0.5$ is 0.8181 (Table 10).

$$L_s = 0.03948\ (400)^2\ \frac{100}{40}\ 0.8181 = 0.03948 \times 400\ 000 \times 0.8181$$
$$= 12\ 919 \text{ microhenries}$$
$$= 0.012919 \text{ henry}$$

$$\begin{aligned} \log 0.03948 &= \bar{2}.59638 \\ \log 400\ 000 &= 5.60206 \\ \log\ \ 0.8181 &= \bar{1}.91281 \\ \hline & \quad 4.11125 \end{aligned}$$

Entering Tables 11 and 12 with $\frac{d}{D} = 0.5$, $n = 400$, we find

$$\begin{aligned} A &= -0.136 \\ B &= \ \ \ 0.335 \\ \hline A+B &= \ \ \ 0.199 \end{aligned}$$

The correction in (155) is, accordingly

$$0.01257\ (400)\ 10\ (0.199) = 9.99 \text{ microhenries.}$$

The total inductance is 12 919 − 10 = 12 909 microhenries.

Example.—A coil of 79 turns of wire of about 0.8 mm bare diameter. The mean diameter is about 22.3 cm and, for determining the pitch, it was found that the distance from the first to the 79th wire was 9.0 cm.

We have, then,

$$a=11.15,\ \ D=\frac{9.0}{78}=0.115,\ \ b=nD=79\times 0.115=9.12$$

$$\frac{2a}{b}=2.445,\ \ \frac{d}{D}=\frac{0.08}{0.115}=0.7$$

The value of K is given by Table 10 as 0.4772, so that

$$L_s=0.03948\ (79)^2\ \frac{(11.15)^2}{9.12}\ 0.4772=1602.8 \text{ microhenries}$$

log 0.03948	$=\bar{2}.59638$
2 log 79	$=3.79526$
2 log 11.15	$=2.09454$
log 0.4772	$=\bar{1}.67870$
	4.16488
log 9.12	$=0.95999$
	3.20489

For $n=79$, $\frac{d}{D}=0.7$, Tables 11 and 12 give

$$A=0.200$$
$$B=0.326$$
$$(A+B)=0.526$$

The correction is $0.01257 \times 79 \times 11.15 \times 0.526 = 5.8$ microhenries, and the total is 1597.0 microhenries. The measured inductance of this coil is 1595.5.

COIL WOUND WITH WIRE OR STRIP OF RECTANGULAR CROSS SECTION

Approximate values may be obtained for a coil wound with rectangular-section wire or strip by using the simple formula (153), as already explained. More precise values for the low-frequency inductance could be calculated in the same manner as for round wire above, using different values for A and B. It is simpler, however, to use formula (156) below, which applies to the single-layer coil if the symbols are given the following meaning: a = radius measured from the axis out to the center of the cross section of the wire; b = the pitch of the winding D, multiplied by the number of turns n; $c=w$ = the radial dimension of the wire; t = the axial thickness of the wire. The correction for the cross section of the wire is obtained by using formulas (161) and (162), using $\nu=\frac{w}{D}$, $\tau=\frac{t}{D}$.

Example.—A solenoid of 30 turns is wound with ribbon ¼ inch by $\frac{1}{16}$ inch thick, with a winding pitch of ¼ inch to form a solenoid of mean diameter 10 inches.

$$\text{Here } a = 5 \times 2.54 = 12.70 \text{ cm},\ w = c = \frac{1}{4}(2.54) = 0.635 \text{ cm}$$

$$b = 30 \times \frac{1}{4}(2.54) = 19.05 \text{ cm},\ c/b = \frac{1}{30},\ D = 0.635$$

$$t = \frac{1}{16}(2.54)$$

for the equivalent coil. Solving this by Rosa's formula (156), using $\frac{2a}{b} = \frac{4}{3}$, $K = 0.6230$ (Table 10), $\frac{b}{c} = 30$, $B_s = 0.3218$, we find $L_u = 182.55\ \mu h$. The value obtained by Stefan's formula (157) is very slightly in error, being 182.5.

To obtain the correction, we have $\nu = \frac{w}{D} = 1$, $\tau = \frac{1}{4}$, and therefore

$$A_1 = \log_e \frac{2}{1.25} = 0.470$$

$$B_1 = -2\left[\frac{29}{30}0.060 + \frac{28}{30}0.018 + \frac{27}{30}0.008 + \frac{26}{30}0.005 + \cdot \cdot \cdot + \frac{21}{30}0.001\right] = -0.188$$

so that the correction is (0.01257) 30 (12.70) (0.282) = 1.35 μh, and the total inductance is 183.9.

INDUCTANCE OF POLYGONAL COILS

Such coils, instead of being wound on a cylindrical form, are wrapped around a frame such that each turn of wire incloses an area bounded by a polygon.

No formula has been developed to fit this case, but it is found that the inductance of such a coil (when the number of sides of the polygon is fairly large) may be calculated, within 1 per cent, by assuming that the coil is equivalent to a helix, whose mean radius is equal to the mean of the radii of the circumscribed and inscribed circles of the polygon. That is, if r = the radius of the circumscribed circle, Fig. 182 (which can be measured without difficulty for a polygon for which the number of sides N is an even number), then the modified radius $a_o = r \cos^2 \frac{\pi}{2N}$ is to be used for a in the formulas (153) and (155) of the preceding section.[a]

[a] For further information regarding polygonal coils reference may be made to Scientific Papers of the Bureau of Standards No. 468, by F. W. Grover, Formulas and Tables for the Calculation of the Inductance of Coils of Polygonal Form, 1923.

Examples.—The following table gives the results obtained by this method for some 12-sided polygonal coils, the measured inductance being given for comparison. For $N=12$, $a_o=0.983r$.

Coil	r	a_o	n	D	b	L_o calculated μh	L_o measured μh
A	6.35	6.24	23	0.32	7.3	63.0	61.7
B	8.25	8.10	28	.32	9.0	124.7	126.3
C	11.43	11.22	52	.212	11.0	638.0	630.5
D	11.43	11.22	34	.318	10.8	274.9	274.6
E	13.97	13.73	64	.211	13.1	1119.5	1115.5
F	19.05	18.71	117	.158	18.5	5389	5387

MULTIPLE-LAYER COILS

Different formulas are used for long than for short coils. For long coils of few layers, sometimes called multiple-layer solenoids, the inductance is given, approximately, by

$$L_u \doteq L_s - \frac{0.01257 n^2 ac}{b}(0.693 + B_s) \tag{156}$$

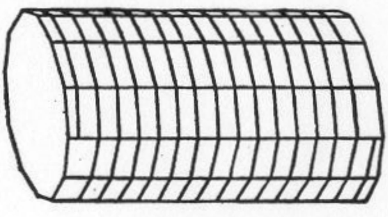

FIG. 182.—*Polygonal coil*

where L_s = inductance, calculated by (153), letting

n = number of turns of the winding

a = radius of coil measured from the axis to the center of cross section of the winding

b = length of coil = distance between centers of turns, times number of turns in one layer

c = radial depth of winding = distance between centers of two adjacent layers times number of layers

B_s = correction given in Table 13, page 284, in terms of the ratio $\frac{b}{c}$

Values obtained by this formula are less accurate as the ratio c/a is greater, and may be a few parts in 1000 in error for values of this ratio as great as 0.25, and $\frac{b}{a}$ as great as 5. For accurate results a correction needs to be applied to L_u(see (159) below).

The solution of the problem for short coils is based on that for the ideal case of a circular coil of rectangular cross section. Such a coil would be realized by a winding of wire of rectangular cross

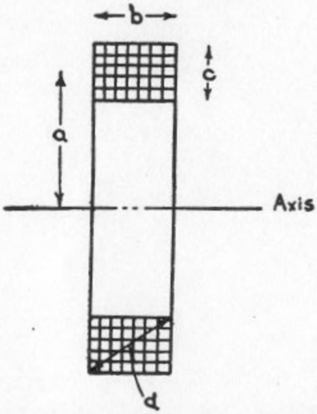

FIG. 183.—*Multiple-layer coil with winding of rectangular cross section*

section, arranged in several layers, with an insulating space of negligible thickness between adjacent wires.

Let a = the mean radius of the winding, measured from the axis to the center of the cross section

b = the axial dimension of the cross section

c = the radial dimension of the cross section

$d = \sqrt{b^2 + c^2}$ = the diagonal of the cross section

n = number of turns of rectangular wire.

Then, if the dimensions b and c are small in comparison with a, the inductance is very accurately given by Stefan's formula, which, for $b > c$, takes the form

$$L_u = 0.01257\, an^2 \left[\left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2} \right) \log_e \frac{8a}{d} - y_1 + \frac{b^2}{16a^2} y_2 \right]$$
$$= 0.01257\, an^2 \left[2.303 \left(1 + \frac{b^2}{32a^2} + \frac{c^2}{96a^2} \right) \log_{10} \frac{8a}{d} - y_1 + \frac{b^2}{16a^2} y_2 \right] \quad (157)$$

where y_1 and y_2 are constants given in Table 14, page 285.

For disk or pancake coils, $b<c$, and the formula becomes

$$L_u = 0.01257\, an^2\left[\left(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2}\right)\log_e\frac{8a}{d}-y_1+\frac{c^2}{16a^2}y_3\right]$$
$$=0.01257\, an^2\left[2.303\left(1+\frac{b^2}{32a^2}+\frac{c^2}{96a^2}\right)\log_{10}\frac{8a}{d}-y_1+\frac{c^2}{16a^2}y_3\right] \quad (158)$$

in which y_1 and y_3 are given in Table 14, page 285.

The constant y_1 is the same function of both b/c and c/b, so that its argument, in any given case, is the ratio of the smaller dimension to the larger; y_2 and y_3 are functions of c/b and b/c, respectively, the arguments being not greater than unity in either case. The error due to the neglect of higher order terms in $\frac{b}{a}$ and $\frac{c}{a}$ in formulas (157) and (158) becomes more important the greater the diagonal of the cross section is, in comparison with the mean radius, but even in the most unfavorable case, c/b small, the inaccuracy with values of the diagonal as great as the mean radius does not exceed one-tenth of 1 per cent. The accuracy is greater with disk coils than with long coils, and best of all when the cross section is square.

For long coils (those in which the length b is greater than the mean radius a), the error of formula (157) becomes rapidly greater. In cases where both dimensions of the cross section are large, in comparison with the mean radius, no formulas well adapted to numerical computations are available, but this is not to be regarded as a case of practical importance in radio engineering.

COIL OF ROUND WIRE WOUND IN A CHANNEL OF RECTANGULAR CROSS SECTION

If we suppose that the distance between the centers of adjacent wires in the same layer is D_1, and that the distance between the centers of wires in adjacent layers is D_2, then the dimensions of the cross section of the equivalent coil with uniform distribution of the current over the cross section will be given by $b=n_1D_1$, $c=n_2D_2$, where n_1 and n_2 are, respectively, the number of turns per layer, and the number of layers.

The inductance of the equivalent coil calculated by formulas (156), (157), or (158), using these dimensions and the same mean radius as the actual coil, is a very close approximation to the value for the actual coil, unless the percentage of the cross section occupied by insulating space is large.[a]

[a] For further information regarding circular coils of rectangular cross section reference may be made to Scientific Papers of the Bureau of Standards No. 455, by F. W. Grover, Tables for the Calculation of the Inductance of Circular Coils of Rectangular Cross Section, 1922.

When such is the case, the correction to the inductance, given in the following formula, may be added:

$$\Delta L = 0.01257\, an\left[2.30 \log_{10} \frac{D}{d} + 0.138 + E\right] \qquad (159)$$

in which D = distance between centers of adjacent wires

d = diameter of the bare wire

E = a term depending on the number of turns and their arrangement in the cross section. Its value may with sufficient accuracy be taken as equal to 0.017. The correction in (159) should, in any case, be roughly calculated, to see if it need be taken into account.

Example.—Suppose a coil of winding channel $b = c = 1.5$ cm, wound with 15 layers of wire, with 15 turns per layer, the mean radius of the winding being 5 cm. Diameter of bare wire = 0.08 cm.

In this case formula (158) gives

$$n = 225,\ d^2 = 4.5,\ \frac{d^2}{a^2} = \frac{4.5}{25} = 0.18,\ b/c = 1,\ y_1 = 0.8483,\ y_2 = 0.816$$

$$L_u = (0.01257)(5)(225)^2\left[\left\{1 + \frac{3(0.3)^2 + (0.3)^2}{96}\right\}\log_e \frac{8}{\sqrt{0.18}} - 0.8483 + \frac{(0.3)^2}{16}0.816\right]$$

$$\begin{array}{llll}
\log 8 = 0.90309 & 2.76310^a & 1.00375 \log_e \frac{8a}{d} = & 2.9478 \\
\frac{1}{2}\log 0.18 = \bar{1}.62764 & .17269 & -y_1 = - & .8483 \\
 & .00104 & & \overline{2.0995} \\
\log_{10}\frac{8a}{d} = 1.27545 & 2.93683 = \log_e \frac{8a}{d} & \frac{0.09}{16}0.816 = & .0046 \\
 & & & \overline{2.104}
\end{array}$$

$$\begin{array}{ll}
\log_{10} 2.104 & = 0.32305 \\
2 \log_{10} 225 & = 4.70436 \\
\log_{10} 0.01257 & = \bar{2}.09934 \\
\log_{10} 5 & = 0.69897 \\
 & \overline{3.82572}
\end{array}$$

$L_u = 6694$ microhenries.

The correction for insulation is found from (159), as follows:

$$\frac{D}{d} = \frac{0.1}{0.08} = \frac{5}{4},\ \log_{10}\frac{5}{4} = 0.09691,\ \log_e \frac{5}{4} = 0.223$$

$$\begin{array}{r}
0.138 \\
E = 0.017 \\
\overline{0.378}
\end{array}$$

correction = (0.01257) (5) (225) 0.378 = 3.34μh

The total inductance is 6697 microhenries = 6.697 millihenries. The correction could, in this case, have been safely neglected.

Example.—A coil of 10 layers of 100 turns per layer, mean radius = 10 cm, the wires being spaced 0.1 cm apart.

For this case $n = 1000$, $a = 10$, $b = 10$, $c = 1$.

Using formula (156) with $\frac{2a}{b} = 2$, $K = 0.5255$, $b/c = 10$

$$L_s = (0.03948)\frac{\overline{1000}^2\,\overline{10}^2}{10}0.5255 = 207\,400 \text{ microhenries.}$$

For the correction, Table 13 gives for $\frac{b}{c} = 10$

$$\begin{array}{r} 0.693 \\ B_s = 0.279 \\ \hline 0.973 \end{array}$$

so that the correction $= (0.01257)10^6\frac{10}{10}0.973 = 12\,200$ and the inductance is

$$L_u = 207\,400 - 12\,200 = 195\,200 \text{ microhenries}$$
$$= 195.2 \text{ millihenries.}$$

The formula (157) gives a value about one part in 900 higher than this.

INDUCTANCE OF A FLAT SPIRAL

Such a spiral may be wound of metal ribbon, or of thicker rectangular wire, or of round wire. In each case, the inductance calculated for the equivalent coil, whose dimensions are measured by the method about to be treated, will generally be as close as 1 per cent to the truth, the value thus computed being too small.

If n wires, Fig. 184, of rectangular cross section are used, whose width in the direction of the axis is w, whose thickness is t, and whose pitch, measured from the center of cross section of one turn to the corresponding point of the next wire is D, then the dimensions of the cross section of the equivalent coil are to be taken as $b = w$, $c = nD$, and as before $d = \sqrt{b^2 + c^2}$.

The mean radius of the equivalent coil is to be taken as $a = a_1 + \frac{1}{2}(n-1)D$, the distance a_1 being one-half of the distance AB (see Fig. 185) measured from the innermost end of the spiral across the center of the spiral to the opposite point of the innermost turn.

The inductance L_u of the equivalent coil is to be calculated using the above dimensions in (158), assuming for n the same number of turns as that of the spiral.

If *round* wire is employed, the same method is used for obtaining the mean radius a and the dimension c, but it is more convenient to take b as zero, and use for the calculation of the inductance of the equivalent coil the special form of (158) which follows when b is placed equal to zero.

$$L_s = 0.01257\, an^2 \left\{ \log_e \frac{8a}{c} - \frac{1}{2} + \frac{c^2}{96a^2}\left(\log_e \frac{8a}{c} + \frac{43}{12}\right)\right\}$$

$$= 0.01257\, n^2 a \left\{ 2.303 \log_{10} \frac{8a}{c} - \frac{1}{2} + \frac{c^2}{96a^2}\left(2.303 \log_{10} \frac{8a}{c} + \frac{43}{12}\right)\right\} \qquad (160)$$

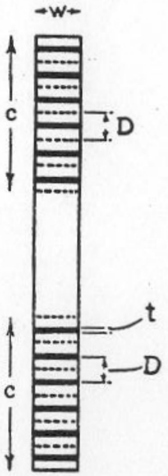

FIG. 184.—*Sectional view of flat spiral wound with metal ribbon*

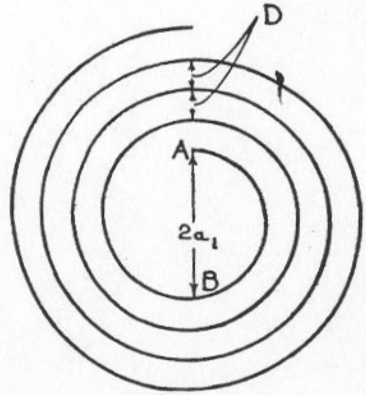

FIG. 185.—*Side view of flat spiral*

The correction for cross section may, in each case, be made by adding 0.01257 na $(A_1 + B_1)$ to the value of inductance for the equivalent coil.

For round wires the quantities A_1 and B_1 may be taken as equal to A and B in the Tables 11 and 12, page 284, just as in the case of single-layer coils of round wire.

In the case of wire or strip of rectangular cross section the matter is more complicated on account of the two dimensions of the cross section.

If we let $\frac{w}{D} = \nu$ and $\frac{t}{D} = \tau$, then the quantities involved in the calculation of A_1 and B_1 may be made to depend on these two parameters alone. The equations are then with sufficient accuracy:

$$A_1 = \log_e \frac{\nu + 1}{\nu + \tau} = 2.303 \log_{10} \frac{\nu + 1}{\nu + \tau} \tag{161}$$

$$B_1 = -2\left[\frac{n-1}{n}\delta_{12} + \frac{n-2}{n}\delta_{13} + \frac{n-3}{n}\delta_{14} + \ldots + \frac{1}{n}\delta_{1n}\right] \tag{162}$$

in which δ_{12}, δ_{13}, etc., are to be taken from Table 15, page 285.

Example.—For a spiral of 38 turns, wound with copper ribbon whose cross sectional dimensions are 3/8 by 1/32 inch, the inner diameter was found to be $2a_1 = 10.3$ cm and the measured pitch was found to be 0.40 cm.

The dimensions of the equivalent coil of rectangular cross section are, accordingly,

$$b = 3/8 \text{ inch} = 0.953 \text{ cm},$$

$$a = \frac{10.3}{2} + \frac{1}{2}\, 37\, (0.4) = 12.55,$$

$$c = 38 \times 0.40 = 15.2.$$

For this coil $b/c = 0.0627$ which gives (Table 14) $y_1 = 0.5604$,

$$y_2 = 0.599, \frac{d^2}{a^2} = 1.472, \log_e \frac{8a}{d} = 1.886.$$

Hence from (158),

$$L_u = (0.01257)\,(12.55)\,(38)^2\,[1.015(1.886) - 0.5604 + 0.055]$$
$$= 323.3 \text{ microhenries.}$$

For this spiral $\nu = 2.38$, $\tau = 0.198$

$$A_1 = 2.303 \log_{10} \frac{3.38}{2.58} = 0.270$$

$$B_1 = -2\left[\frac{37}{38}(0.028) + \frac{36}{38}(0.013) + \frac{35}{38}(0.007) + \frac{34}{38}(0.004) + \frac{33}{38}(0.003) + \frac{32}{38}(0.002) + \frac{31}{38}(0.002) + \frac{30}{38}(0.001) + \cdots\right]$$

$$= -0.112, \quad A_1 + B_1 = 0.159$$

and the total correction is (0.01257) (38) (12.55) (0.159) = 0.95 μh so that the total inductance of the spiral is 324.2 microhenries. The measured value was 323.5.

INDUCTANCE OF A SQUARE COIL

Two cases present themselves

(*a*) A square coil wound in a rectangular cross section.

(*b*) A square coil wound in a single layer.

MULTIPLE-LAYER SQUARE COIL

Let a be the side of the square measured to the center of the rectangular cross section which has sides b and c, and let n be the total number of turns.

Then

$$L_u = 0.008\, an^2 \left[2.303 \log_{10} \frac{a}{b+c} + 0.2235 \frac{b+c}{a} + 0.726 \right] \qquad (163)$$

If the cross section is a square, $b = c$, this becomes

$$L_u = 0.008\, an^2 \left[2.303 \log_{10} \frac{a}{b} + 0.447 \frac{b}{a} + 0.033 \right] \qquad (164)$$

A correction for the insulating space between the wires may be calculated by equation (159) if we replace $0.01257\, an$ therein by

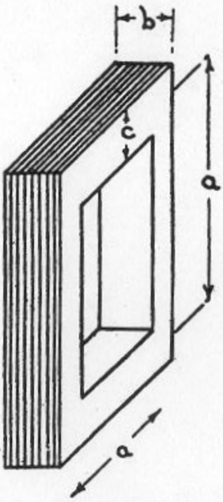

FIG. 186.—*Multiple-layer square coil with winding of rectangular cross section*

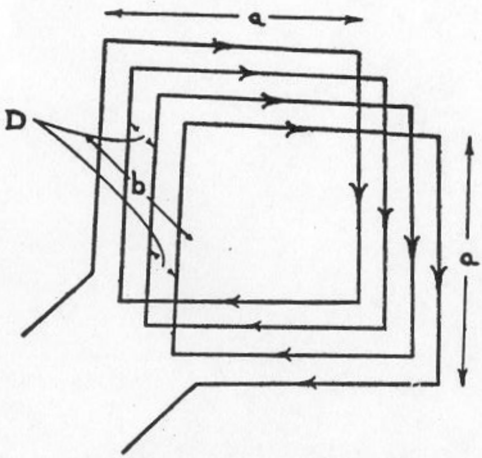

FIG. 187.—*Single-layer square coil*

$0.008\, an$. This correction is additive, but will be negligible unless the insulating space between the wires is large.

SINGLE-LAYER SQUARE COIL

Let a = the side of the square, measured to the center of the wire
n = number of turns
D = pitch of the winding, that is, the distance between the center of one wire and the center of the next (Fig. 187)
$b = nD$ (See p. 252.)

Then

$$L_0 = 0.008\ an^2\left[2.303\ \log_{10}\frac{a}{b} + 0.726 + 0.2231\ \frac{b}{a}\right] - 0.008\ an\ [A + B] \qquad (165)$$

in which A and B are constants having the same meaning as in (155) to be taken from Tables 11 and 12, if the wires are of round cross section. If the wire is a rectangular strip having a dimension t along the axis of the coil and w perpendicular to it, calculate L_u by (163) and correct for cross section by (161) and (162) and Table 15, using $0.008\ an\ (A_1 + B_1)$.

Example.—Suppose a square coil, 100 cm on a side, wound in a single layer with 4 turns of round wire, 0.1 cm bare diameter, the winding pitch being 0.5 cm.

In this case $n = 4$ $\quad d = 0.1 \quad b = 4 \times 0.5 = 2.0$

$a = 100 \quad D = 0.5$

The main term in formula (165) gives

$$0.008 \times 100 \times 16\left[2.303\ \log_{10}\frac{100}{2} + 0.726 + 0.004\right]$$
$$= 12.8\ [3.912 + 0.726 + 0.004] = 59.42 \text{ microhenries}$$

Entering Tables 11 and 12, page 284, with $\frac{d}{D} = \frac{0.1}{0.5} = 0.2$ and $n = 4$,

$$\begin{aligned} A &= -1.053 \\ B &= 0.197 \\ \hline \text{sum} &= -0.856 \end{aligned}$$

$0.008\ an\ [-0.856] = -2.74$ microhenries,
so that $L_u = 59.42 + 2.74 = 62.16$ microhenries.

This result may be checked by computing the self-inductance L_1 of a single turn and the mutual inductances M_{pq} of the individual turns, and summing them up.

Thus we find

$$\begin{aligned} 4\,L_1 &= 22.65 \\ 6M_{12} &= 21.74 \\ 4M_{13} &= 12.29 \\ 2M_{14} &= 5.50 \\ \hline & \,62.18 \text{ microhenries.} \end{aligned}$$

Formula (165) applies only when the length b is small compared with the side of the square a.

RECTANGULAR COIL OF RECTANGULAR CROSS SECTION

Let the sides of the rectangle be a and a_1, the dimensions of the cross section b and c, and the number of turns n, $g=\sqrt{a^2+a_1^2}$

$$L_u=0.00921\,(a+a_1)\,n^2\left[\log_{10}\frac{2aa_1}{b+c}-\frac{a}{a+a_1}\log_{10}(a+g)-\frac{a_1}{a+a_1}\log_{10}(a_1+g)\right]+0.004\,(a+a_1)\,n^2\left[2\left(\frac{g}{a+a_1}\right)-\frac{1}{2}+0.447\frac{(b+c)}{(a+a_1)}\right] \qquad (166)$$

Correct for cross section by (159) for round wire.

SINGLE-LAYER RECTANGULAR COIL

Let a and a_1 be the sides of the rectangle, D the pitch of the winding, $b=nD$, and n the number of turns. Then

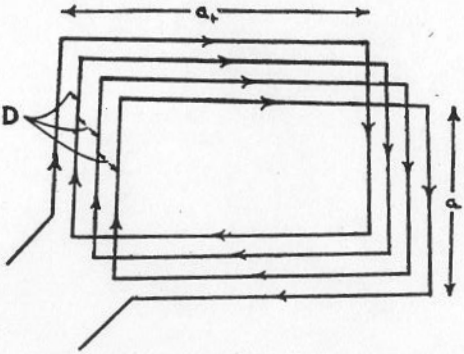

FIG. 188.—*Single-layer rectangular coil.*

$$L_o=0.00921\,(a+a_1)\,n^2\left[\log_{10}\frac{2aa_1}{b}-\frac{a}{a+a_1}\log_{10}(a+g)-\frac{a_1}{a+a_1}\log_{10}(a_1+g)\right]+0.004\,(a+a_1)\,n^2\left[\frac{2g}{a+a_1}-\frac{1}{2}-0.447\,\frac{b}{a+a_1}\right]-0.004\,(a+a_1)\,n\,(A+B) \qquad (167)$$

error see p.2

where A and B are to be taken from Tables 11 and 12, if the coil is wound with round wire. If wound with strip, take $b=nD$ and c = radial thickness of strip. Calculate L_u by (166) and correct for cross section by (161), (162), and Table 15.

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FLAT RECTANGULAR COIL

Let a_0 and a'_0 be the outside dimensions of the coil, measured between centers of the wire, D the pitch of the winding, measured between the centers of adjacent wires (Fig. 189), n the number of complete turns, d the diameter of the bare wire, $c = nD$.

$g = \text{diagonal} = \sqrt{a^2 + a_1^2}$, $a = a_0 - (n-1)D$, $a_1 = a'_0 - (n-1)D$.

Then

$$L_0 = L_u - 0.004\ n(a + a_1)(A + B)$$

where

$$L_u = 0.009210\ n^2\left[(a + a_1)\log_{10}\frac{2aa_1}{c} - a\log_{10}(a + g) - a_1\log_{10}(a_1 + g)\right] + 0.004\ n^2\left[2g - \frac{a + a_1}{2} + 0.447\ c\right] \quad (168)$$

and A and B are constants to be taken from Tables 11 and 12 for round wire. If the coil is wound with rectangular strip, put b = width of the strip, and $c = nD$, and calculate L_u by (166) using for A and B the values A_1 and B_1 of (161) and (162) Table 15.

FLAT SQUARE COIL

If a_0 be here the side of the square, measured between centers of two outside wires, and $a = a_0 - (n-1)D$, the nomenclature being as in the previous section,

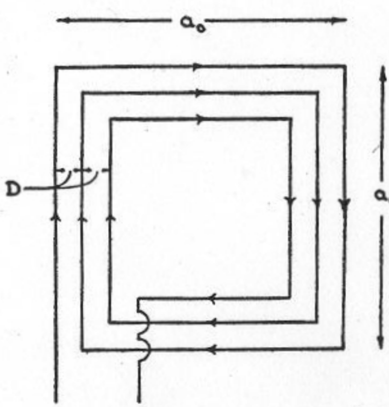

FIG. 189.—*Flat square coil.*

$$L_0 = L_u - 0.008\ n\ a\ (A + B)$$

in which

$$L_u = 0.008\ n^2 a\left[2.303\ \log_{10}\frac{a}{c} + 0.2235\frac{c}{a} + 0.726\right] \quad (169)$$

For round wire the constants A and B are given in Tables 11 and 12. If the coil is wound with strip proceed as for rectangular flat coils of strip, above.

Example.—A coil of 4 turns of 0.22 cm stranded wire was found to have $a_0 = 102$ cm, the pitch of the winding being $D = 2.25$ cm. Here

$$a = 102 - 3 \times 2.25 = 95.25$$
$$c = 4 \times 2.25 = 9.0$$

$$L_u = 0.008 \times 16 \times 95.25 \left[2.303 \log_{10} \frac{95.25}{9.0} + 0.2235 \frac{9.0}{95.25} + 0.726 \right]$$
$$= 16 \times 0.762 \ [2.359 + 0.021 + 0.726] = 37.87 \ \mu h$$

For

$$n = 4 \text{ and } \frac{d}{D} = \frac{0.22}{2.25} = 0.098, \text{ Tables 11 and 12 give}$$
$$A = -1.767, \text{ and } B = 0.197$$

the correction is $0.008 \times 4 \times 95.25 \ (-1.570) = -4.79 \ \mu h$ so that $L_0 = 37.87 + 4.79 = 42.66$ microhenries.

The measured value, uncorrected for lead wires was 44.5 microhenries.

DOUBLE FLAT RECTANGULAR COIL

Such a coil consists of two similar flat, rectangular coils, such as are treated in the preceding sections, placed with their axes in the same straight line, and their planes at a distance x apart. The two sections of such a coil may be used either singly, or in series, or in parallel.

The general method of treatment is to obtain the inductance L_1 of the single sections by formula (168) or (166), as described in the preceding sections, and the mutual inductance of the two sections, as shown below.

Then when used in series $L' = 2(L_1 + M)$, and when used in parallel $L'' = \frac{L_1 + M}{2}$.

To obtain the mutual inductance, formula (183) or (184) for two equal, parallel rectangles or squares, multiplied by the product of the number of turns of the two, should be used, putting for the dimensions of the rectangles a and a_1 as defined under (168) and (169) and for the distance D in (183) or (184) a modified distance r given by the expression

$$r = kc, \ c = nD, \ (x/c \text{ small})$$

in which

$$2.303 \log_{10} k = 2.303 \frac{x^2}{c^2} \log_{10} \frac{x}{c} + \pi \frac{x}{c} - \frac{3}{2} - \frac{3}{2} \frac{x^2}{c^2} - \frac{1}{12} \frac{x^4}{c^4} \quad (170)$$

When x is not small in comparison with c, r will have to be calculated by the equation

$$\log_{10} r = \frac{x^2}{c^2} \log_{10} x + \frac{1}{2}\left(1 - \frac{x^2}{c^2}\right) \log_{10} (c^2 + x^2) + \frac{\left(2\frac{x}{c}\tan^{-1}\frac{c}{x} - \frac{3}{2}\right)}{2.303} \quad (171)$$

When the distance x between the planes of the coils is chosen equal to the pitch D of their windings, the calculation of their inductance, when joined in series, may be obtained in a simpler manner. Putting $b = 2D$ and $n_1 = 2n$, the number of turns of the two windings in series,

$$L' = 0.008\, n_1^2 a \left[2.303 \log_{10} \frac{a}{b+c} + 0.2235 \frac{b+c}{a} + 0.726\right]$$
$$+ 0.008\, n_1 a \left[2.303 \log_{10} \frac{D}{d} + 0.153\right] \quad (172)$$

for a square coil, and

$$L' = 0.009210\, n_1^2 \left[(a + a_1) \log_{10} \frac{2aa_1}{b+c} - a \log_{10} (a+g)\right.$$
$$\left. - a_1 \log_{10} (a_1 + g)\right] + 0.004\, n_1^2 \left[2g - \frac{a+a_1}{2} + 0.447(b+c)\right]$$
$$+ 0.004\, n_1 (a + a_1) \left[2.303 \log_{10} \frac{D}{d} + 0.153\right] \quad (173)$$

for a rectangular coil

$$g = \sqrt{a^2 + a_1^2}, \ d = \text{diameter of bare wire.}$$

Example.—As an example of the use of these formulas, take the case of an actual coil of two sections, each being a flat, square coil of 5 turns of 0.12 cm wire, wound with a pitch of $D = 1.27$ cm, the distance of the planes of the coils being $x = 1.27$ cm. The length of a side of the outside turn was 101 cm.

Putting $n = 5$, $a = 101 - 4 \times 1.27 = 95.9$, $c = 5 \times 1.27 = 6.35$, and $d/D = 0.1$, formula (169) gives $L_1 = 66.28 + 6.14 = 72.42 \mu h$, for a single section.

To obtain the mutual inductance, we find by (170) for

$$\frac{x}{c}=\frac{1.27}{6.35}=0.2$$

$$2.303 \log_{10} k = 2.303\times 0.04\,(-0.699) + 0.2\,\pi - \frac{3}{2} - \frac{3}{2}(0.04) - \frac{1}{12}(0.0016)$$

$$= -0.0644 + 0.6283 - 1.5 - 0.06 - 0.0001$$

$$= -0.9962$$

$$\log_{10} k = -0.4326 = \bar{1}.5674$$

$$k = 0.3693 \text{ and } r = 0.3693 \times 6.35 = 2.344$$

Putting this value of r in place of D in (184) with $a=95.9$

$$M = 0.008\times 5\times 5\left[2.303\times 95.9 \log_{10}\left(\frac{191.8\times 95.93}{231.5\times 2.344}\right) + 135.62 - 191.86 + 2.34\right] = 56.82\ \mu h$$

For the two coils in series, then

$$L' = 2(72.42 + 56.82) = 258.5\ \mu h$$

and for the parallel arrangement

$$L'' = \frac{72.42+56.82}{2} = 64.6\ \mu h$$

The inductance of the coils in series may also be found by putting $a=95.9$, $b=6.35$, $c=2.54$, $n_1=10$ in (163) and (159) and we find $L=239.8+18.8=258.6\ \mu h$ in agreement with the other method.

69. MUTUAL INDUCTANCE

The following formulas for mutual inductance hold strictly only for low frequencies. In general, however, the values will be the same at high frequencies.

TWO PARALLEL WIRES OR BARS SIDE BY SIDE

Let l = length of each wire or bar.
D = distance between centers of the wires.

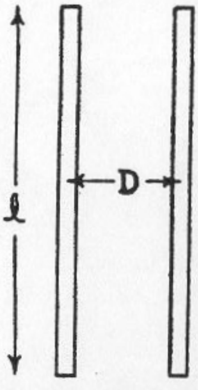

Fig. 190.—*Two parallel wires side by side*

The following expression is exact when the wires have no appreciable cross section, but is sufficiently exact even when the cross section is large if l is great compared with D. Within these limits the shape is immaterial.

$$M = 0.002\left[2.303\, l \log_{10} \frac{l+\sqrt{l^2+D^2}}{D} - \sqrt{l^2+D^2} + D\right] \tag{174}$$

$$= 0.002 l \left[2.303 \log_{10} \frac{2l}{D} - 1 + \frac{D}{l}\right] \text{nearly.} \tag{175}$$

TWO WIRES END TO END WITH THEIR AXES IN LINE

Let the lengths of the two wires be l and m, their radii being supposed to be small. Then,

$$M = 0.002303 \left[l \log_{10} \frac{l+m}{l} + m \log_{10} \frac{l+m}{m}\right] \tag{176}$$

FIG. 191.—*Two wires end to end in same straight line*

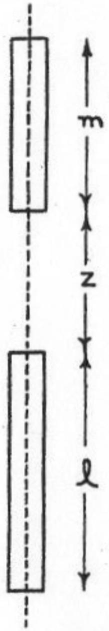

FIG. 192.—*Two wires in same straight line but separated*

TWO WIRES WITH THEIR AXES IN THE SAME STRAIGHT LINE BUT SEPARATED

Let their lengths be l and m and the distance between the nearer ends be Z.

$$M = 0.002303\, [(l+m+Z) \log_{10} (l+m+Z) + Z \log_{10} Z - (l+Z) \log_{10} (l+Z) - (m+Z) \log_{10} (m+Z)] \tag{177}$$

TWO WIRES WITH AXES IN PARALLEL LINES

If AD, AD', AC, AC', etc., represent the distances shown in the figure, the general formula is

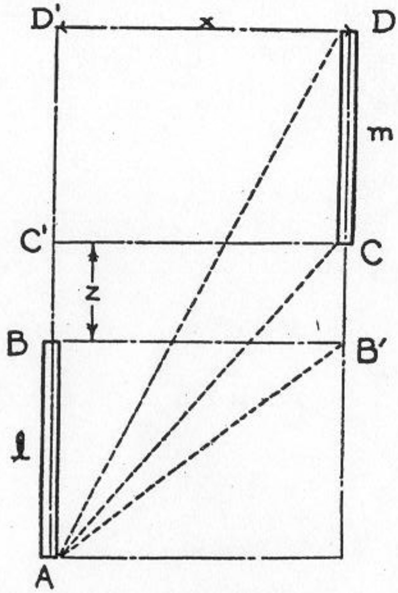

FIG. 193.—*Two wires with axes in parallel lines*

$$\left.\begin{aligned} M = 0.001151 \Bigg[& l \log_{10}\left\{\frac{AD+AD'}{AD-AD'} \times \frac{AC-AC'}{AC+AC'}\right\} \\ & + m \log_{10}\left\{\frac{AD+AD'}{AD-AD'} \times \frac{BD-BD'}{BD+BD'}\right\} \\ & + Z \log_{10}\left\{\frac{AD+AD'}{AD-AD'} \times \frac{AC-AC'}{AC+AC'} \times \frac{BD-BD'}{BD+BD'} \times \frac{BC+BC'}{BC-BC'}\right\}\Bigg] \\ & - 0.001\ (AD-AC-BD+BC) \end{aligned}\right\} \quad (178)$$

the distances being $AD' = l+m+Z$, $AD = \sqrt{x^2+(l+m+Z)^2}$, etc. This formula holds for $Z=0$, but not when one wire overlaps on the other.

When they overlap, as in Fig. 194,

$$M = M_{1,34} + M_{23} + M_{24} \quad (179)$$

in which $M_{1,34}$ is to be calculated by the general formula, using $Z=0$ and putting the segment PV for l and ST for m, while for M_{24} the length VR is put for l and WT for m with $Z=0$. The mutual inductance M_{23} of the overlapping portions is obtained by (174).

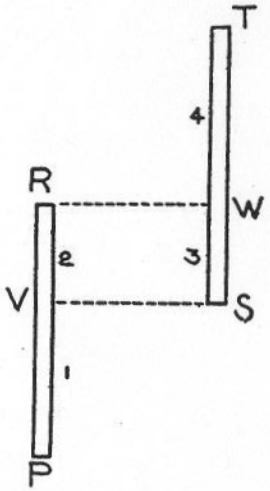

Fig. 194.—*Two wires with axes in parallel lines; a particular case of Fig. 193*

Special Cases.—For the case shown in Fig. 195

$$M=0.001\left[2.303l\log_{10}\left(\frac{l+\sqrt{D^2+l^2}}{D}\right)+D-\sqrt{D^2+l^2}\right] \quad (180)$$

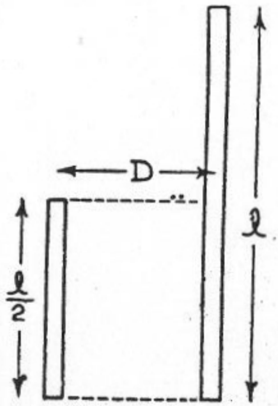

Fig. 195.—*Two wires with axes in parallel lines; another particular case of Fig. 193*

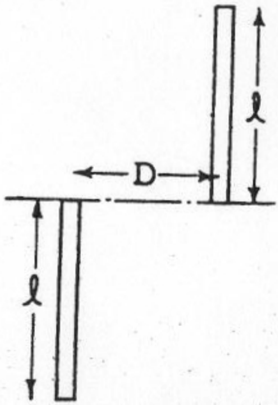

Fig. 196.—*Two wires with axes in parallel lines, with one end of each on the same perpendicular*

and for the wires of Fig. 196

$$M=0.001\left[4.605l\log_{10}\left(\frac{2l+\sqrt{D^2+4l^2}}{l+\sqrt{D^2+l^2}}\right)-\sqrt{D^2+4l^2} + 2\sqrt{D^2+l^2}-D\right] \quad (181)$$

MUTUAL INDUCTANCE OF TWO PARALLEL SYMMETRICALLY PLACED WIRES

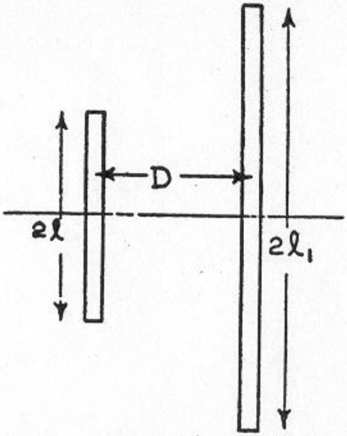

FIG. 197.—*Two parallel symmetrically placed wires*

Putting for the lengths of the two wires $2l$ and $2l_1$ ($2l$ the shorter) and for their distance apart D

$$M = 0.002\left[2.303(2l)\log_{10}\left\{\frac{l+l_1+\sqrt{(l+l_1)^2+D^2}}{D}\right\} + 2.303(l_1+l)\log_{10}\left\{\frac{l+l_1+\sqrt{(l+l_1)^2+D^2}}{l_1-l+\sqrt{(l_1-l)^2+D^2}}\right\} + \sqrt{(l_1-l)^2+D^2} - \sqrt{(l+l_1)^2+D^2}\right] \quad (182)$$

error see p.2

TWO EQUAL PARALLEL RECTANGLES

Let a and a_1 be the sides of the rectangles and D the distance between their planes, the centers of the rectangles being in the same line, perpendicular to these planes

$$M = 0.009210\left[a\log_{10}\left\{\frac{a+\sqrt{a^2+D^2}}{a+\sqrt{a^2+a_1^2+D^2}}\times\frac{\sqrt{a_1^2+D^2}}{D}\right\} + a_1\log_{10}\left\{\frac{a_1+\sqrt{a_1^2+D^2}}{a_1+\sqrt{a^2+a_1^2+D^2}}\times\frac{\sqrt{a^2+D^2}}{D}\right\}\right] + 0.008\left[\sqrt{a^2+a_1^2+D^2} - \sqrt{a^2+D^2} - \sqrt{a_1^2+D^2} + D\right] \quad (183)$$

TWO EQUAL PARALLEL SQUARES

If a is the side of each square and D is the distance between their planes, then the preceding formula becomes

$$M = 0.01842\left[a \log_{10}\left\{\frac{a+\sqrt{a^2+D^2}}{a+\sqrt{2a^2+D^2}} \times \frac{\sqrt{a^2+D^2}}{D}\right\}\right] \qquad (184)$$
$$+0.008\left[\sqrt{2a^2+D^2} - 2\sqrt{a^2+D^2} + D\right]$$

MUTUAL INDUCTANCE OF TWO RECTANGLES IN THE SAME PLANE WITH THEIR SIDES PARALLEL

$$M = (M_{16}+M_{38}+M_{45}+M_{27}) - (M_{18}+M_{25}+M_{36}+M_{47}) \qquad (185)$$

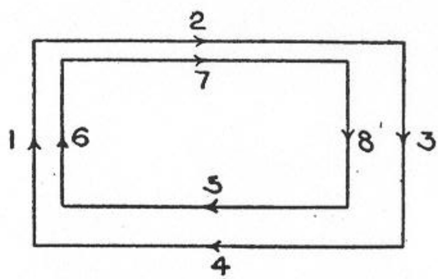

FIG. 198.—*Two rectangles in the same plane with their sides parallel*

the separate mutual inductances being calculated by formula (182), if the sides are symmetrically placed, and by (182) and (178) if that is not the case.

If the rectangles have a common center $M_{16}=M_{38}$, $M_{45}=M_{27}$, $M_{18}=M_{36}$, $M_{25}=M_{47}$ and for the case of concentric squares, we have

$$M = 4(M_{16} - M_{18}) \qquad (186)$$

TWO PARALLEL COAXIAL CIRCLES

This is an important case because of its applicability in calculating the mutual inductances of coils (see below).

Let a = the smaller radius (Fig. 199).

A = the larger radius.

D = the distance between the planes of the circles.

Then

$$\frac{r_2}{r_1} = \sqrt{\frac{\left(1-\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}}$$

must be calculated, and

$$M = F\sqrt{Aa} \tag{187}$$

where F may be obtained by interpolation in Table 16 for the calculated value of $\frac{r_2}{r_1}$ (p. 286).

r_1 = the longest distance between the circumferences.

r_2 = the shortest distance between the circumferences.

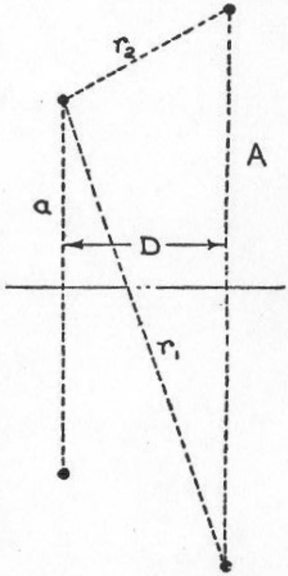

FIG. 199.—*Cross sections of two parallel coaxial circles*

TWO COAXIAL CIRCULAR COILS OF RECTANGULAR CROSS SECTION

If the coil windings are of square, or nearly square, cross section, a first approximation to the mutual inductance is

$$M = n_1 n_2 M_0 \tag{188}$$

where n_1 and n_2 are the number of turns on the two coils and M_0 is the mutual inductance of two coaxial circles, one located at the center of the cross section of one of the coils and the other at the center of the cross section of the other.

Thus, if

a = mean radius of one coil, measured from the axis to the center of cross section,

A = mean radius, similarly measured, of the other coil,

D = distance between the planes passed through the centers of cross section of the coils, perpendicular to their common axis (Fig. 200).

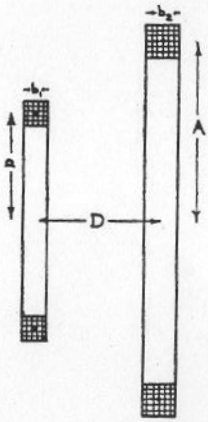

FIG. 200.—*Two parallel coaxial coils with windings of rectangular cross section*

the value M_0 will be computed by formula (187) and Table 16, using the values of a, A, and D, just defined.

If the cross sections of the windings are square, this value will not be more than a few parts in a thousand in error, even with relatively large cross sectional dimensions, except when the coils are close together.

A more accurate value for coils of square cross section may be obtained by supposing the two parallel circles to remain at the distance D, but to have radii

$$a_1 = a\left(1 + \frac{b_1^2}{24\, a^2}\right) \text{ and } A_1 = A\left(1 + \frac{b_2^2}{24\, A^2}\right) \tag{189}$$

where b_1 and b_2 are the dimensions of the square cross sections corresponding to the coils of mean radius a and A, respectively.

When the correction factors in (189) are only a few parts in 1000, the values of r_2/r_1, and hence F, are very little affected, and the fractional correction to the mutual inductance, to allow for the cross sections, is approximately equal to the geometric mean of the fractional corrections to a and A, so that an estimate of the magnitude of the correction to the mutual inductance may be gained with little labor.

With rectangular cross sections the error from the assumption that the coils may be replaced by equivalent filaments at the center of the cross section is more important than in the case of coils of square cross section and rapidly increases as the axial dimension of one or both of the cross sections is increased, in relation to the distance D between the median planes. The error may, easily, be as great as 1 per cent or more in practical cases.

An estimate of the magnitude of the error, in any case, may be made by dividing the coils up into two or more sections of, as nearly as possible, square cross section, and assuming that each portion of the coil may be replaced by a circular filament at the center of its cross section.

Suppose that coil A is divided into two equal parts, and replaced by two filaments 1, 2, while coil B is likewise replaced by two filaments 3, 4, then, assuming that each filament is associated with a number of turns which is the same fraction of the whole number of turns in the coil as the area of the section is to the whole cross sectional area (one-half in this case) we have

$$\begin{aligned} M &= \frac{n_1}{2}\,\frac{n_2}{2}\, M_{13} + \frac{n_1 n_2}{4}\, M_{14} + \frac{n_1 n_2}{4}\, M_{23} + \frac{n_1 n_2}{4}\, M_{24} \\ &= n_1 n_2 \left(\frac{M_{13} + M_{14} + M_{23} + M_{24}}{4}\right) \end{aligned} \tag{190}$$

in which M_{13} is the mutual inductance of the two circular filaments 1 and 3, etc.

For a discussion of more accurate methods for correcting for the cross section of coils, the reader is referred to Bulletin, Bureau of Standards, **8**, pages 33–43; 1912.

If the coils are of the nature of solenoids of few layers, it is best to use the formulas for the mutual inductance of coaxial solenoids given in the next section.

Example.—Suppose two coils of square cross section 2 cm on a side, the radii being, $a=20$, $A=25$, and the distance between their median planes being $D=10$ cm (Fig. 201). Further, suppose that one coil has 100 turns and the other 500.

Then

$$\frac{r_2}{r_1}=\sqrt{\frac{\left(1-\frac{20}{25}\right)^2+\left(\frac{10}{25}\right)^2}{\left(1+\frac{20}{25}\right)^2+\left(\frac{10}{25}\right)^2}}=\sqrt{\frac{0.20}{3.40}}=0.24253$$

From Table 16 we find, corresponding to this value of $\frac{r_2}{r_1}$,

$F=0.01113$. Therefore, from (187)

$$M_0=0.01113\sqrt{25\times 20}=0.2489\mu h$$

and

$$M=n_1n_2M_0=100\times 500\times 0.2489$$
$$=12\ 445 \text{ microhenries}$$
$$=0.012445 \text{ henry.}$$

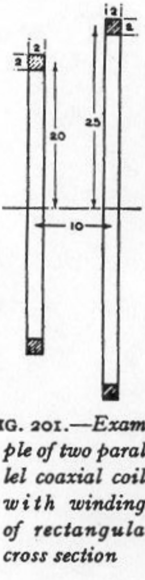

FIG. 201.—*Example of two parallel coaxial coils with windings of rectangular cross section*

If we take account of the cross sections we have from (189)

$$a_1=20\left(1+\frac{2^2}{24\times 20^2}\right)=20\ (1.00042)$$

$$A_1=25\left(1+\left(\frac{2}{25}\right)^2\frac{1}{24}\right)=25\ (1.00027)$$

so that the correction factor to the mutual inductance will be of the order of about $\sqrt{1.00042\times 1.00027}$, or the mutual inductance should be increased by about 3.5 parts in 10 000 only.

Example.—Fig. 202 shows two coils of rectangular cross section. For coil P, $a=20$, $b_1=2$, $c_1=3$, $n_1=600$. For coil Q, $A=25$, $b_2=4$, $c_2=1$, $n_2=400$ and $D=10$. If, first, we replace each coil by a circular filament at the center of its cross section, we have the same value of M_0 as in the previous example, and

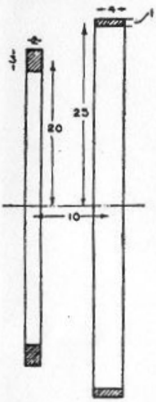

FIG. 202.—*Another example of Fig. 200*

$$M = 600 \times 400 \times 0.2489 \text{ microhenries.}$$

More precise formulas, involving a good deal of computation, show that the true value is

$$M = 600 \times 400 \times 0.249844,$$

so that the approximate value is about 3.8 parts in 1000 too small.

Each coil is then subdivided into two sections and filaments p, q, r, s, imagined to pass through the center of cross section of each of these subdivisions: The data for these filaments are as follows:

Radius	Filaments	a	A	D	r_2/r_1	F
p 19.25	pr	19.25	25	9	0.2365	0.01140
q 20.75	ps	19.25	25	11	.2722	.009872
r 25	qr	20.75	25	9	.2135	.01255
s 25	qs	20.75	25	11	.2506	.01077

We find then

$$M = 600 \times 400 \left\{ \frac{0.2501 + 0.2166 + 0.2858 + 0.2452}{4} \right\} = 600 \times 400 \times 0.24942$$

a result which is 1.7 in 1000 too small.

The increase in accuracy is hardly commensurate with the increased labor.

MUTUAL INDUCTANCE OF COAXIAL SOLENOIDS NOT CONCENTRIC

Gray's formula, given for this case, supposes that each coil approximates the condition of a continuous thin winding, that is, a current sheet.

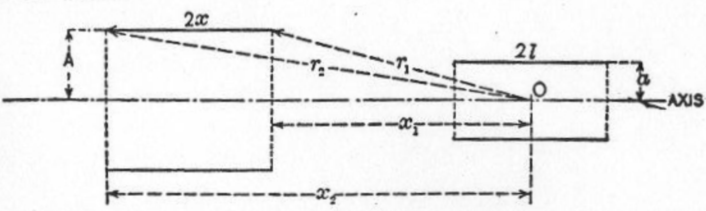

FIG. 203.—*Coaxial solenoids not concentric*

Let a = the smaller radius, measured from the axis of the coil to the center of the wire

A = the larger radius, measured in the same way

$2l$ = length of the coil of smaller radius = number of turns times the pitch of winding

$2x$ = length of the coil of larger radius, measured in the same way

n_1 and n_2 = total number of turns on the two coils

D = axial distance between centers of the coils

$x_1 = D - x \qquad r_1 = \sqrt{x_1^2 + A^2}$

$x_2 = D + x \qquad r_2 = \sqrt{x_2^2 + A^2}$

Then

$$M = 0.009870 \frac{a^2 A^2 n_1 n_2}{2x \cdot 2l}\left[K_1 k_1 + K_3 k_3 + K_5 k_5\right] \qquad (191)$$

in which

$$K_1 = \frac{2}{A^2}\left(\frac{x_2}{r_2} - \frac{x_1}{r_1}\right),\ k_1 = 2l$$

$$K_3 = \frac{1}{2}\left(\frac{x_1}{r_1^5} - \frac{x_2}{r_2^5}\right),\ k_3 = a^2 l\left(3 - 4\frac{l^2}{a^2}\right)$$

$$K_5 = -\frac{A^2}{8}\left[\frac{x_1}{r_1^9}\left(3 - 4\frac{x_1^2}{A^2}\right) - \frac{x_2}{r_2^9}\left(3 - 4\frac{x_2^2}{A^2}\right)\right]$$

$$k_5 = a^4 l\left(\frac{5}{2} - 10\frac{l^2}{a^2} + 4\frac{l^4}{a^4}\right)$$

This formula is most accurate for short coils with relatively great distance between them. In the case of long coils it is sometimes necessary to subdivide the coil into two or more parts. The mutual inductance of each of these parts on the other coil having been found, the total mutual inductance is obtained by adding these values.

Example.—

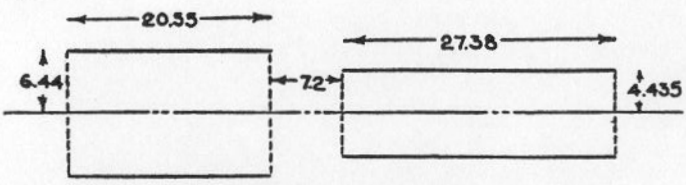

FIG. 204.—*Example of coaxial solenoids not concentric*

$2x = 20.55$	$A = 6.44$	$n_1 = 15$
$2l = 27.38$	$a = 4.435$	$n_2 = 75$

Distance between the adjacent ends of the two solenoids = 7.2 cm.

Then

$$x_1 = 20.89 \qquad\qquad k_1K_1 = 0.04294$$
$$x_2 = 41.44 \qquad\qquad k_3K_3 = .01827$$
$$k_5K_5 = .00519$$
$$\overline{0.06640}$$

and $M = 0.009870\left(\frac{a^2A^2n_1n_2}{2x\ \ 2l}\right)0.06640 = 1.069$ microhenries

$$\begin{aligned}
\log 0.009870 &= \bar{3}.99432 \\
2 \log a &= 1.29378 \\
2 \log A &= 1.61778 \\
\log n_1n_2 &= 3.05115 \\
\log 0.06640 &= \bar{2}.82217 \\
& \overline{2.77920} \\
& 2.75024 \\
& \overline{0.02896} = \log M
\end{aligned}
\qquad
\begin{aligned}
\log 2x &= 1.31281 \\
\log 2l &= 1.43743 \\
& \overline{2.75024}
\end{aligned}$$

Dividing the longer coil into two sections C and D of 37 and 38 turns, respectively, and repeating the calculation for the mutual inductance of these sections on the other coil R (Fig. 204),

For M_{RC}

$$k_1K_1 = 0.04889$$
$$k_3K_3 = .00652$$
$$k_5K_5 = .00005$$
$$\overline{0.05546}$$

For M_{RD}

$$k_1K_1 = 0.01155$$
$$k_3K_3 = .00061$$
$$\overline{0.01216}$$

and $M = M_{RC} + M_{RD} = 0.8917 + 0.1956 = 1.087\ \mu h$.

Further subdivision showed that this last value is not in error by more than 5 parts in 10 000.

The criterion as to the necessity of subdivision is the rapidity with which the terms k_1K_1, k_3K_3, etc., fall off in value. In the first case k_7K_7 and k_9K_9 are not negligible. The expressions for these quantities are not here given because they are laborious to calculate, and it is easier to obtain the value of the mutual inductance by the subdivision method.

COAXIAL, CONCENTRIC SOLENOIDS (OUTER COIL THE LONGER)

The formula here given holds, strictly, only for current sheets. The lengths of the coils should be taken as equal to the number of turns times the pitch of the winding in each case. Then the mutual inductance of the current sheets is not appreciably different from that of the coils.

Let a = smaller radius
A = larger radius
$2x$ = equivalent length of outer coil
$2l$ = equivalent length of inner coil
$g=\sqrt{x^2+A^2}$ = diagonal.

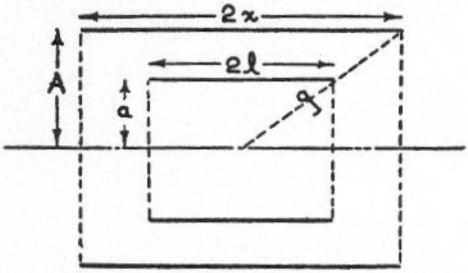

FIG. 205.—*Coaxial concentric solenoids, outer coil being longer*

Then

$$M=\frac{0.01974\ a^2n_1n_2}{g}\left[1+\frac{A^2a^2}{8g^4}\left(3-4\frac{l^2}{a^2}\right)\right] \qquad (192)$$

This formula is more accurate, the shorter the coils and the greater the difference of their radii, but in most practical cases the accuracy is ample. In many cases the second term in (192) is negligible, and it is a good plan to make a preliminary rough calculation of this term to see whether it will need to be considered. In the case of long coils, and of coils of nearly equal radii, the terms neglected in this formula may be as great as 1 per cent. A criterion of rapid convergence is, in general, the smallness of $\frac{a^2A^2}{g^4}$, but the magnitude of the coefficient $\left(3-4\frac{l^2}{a^2}\right)$ and the corresponding coefficients of terms neglected in (192) may in some cases modify this condition for rapid convergence materially.

Example.—

$$2x=30 \qquad 2l=5 \qquad g=\sqrt{250} \qquad \frac{a^2A^2}{g^4}=\frac{4}{625}$$
$$A=5 \qquad a=4$$
$$n_1=300 \qquad n_2=200$$

$$0.01974\ \frac{a^2n_1n_2}{g}=1198.5$$

$$M=1198.5\ (1+.00115)=1199.9 \text{ microhenries.}$$

For the case, however, where

$$2x=30 \qquad a=2 \qquad n_1=300$$
$$2l=24 \qquad A=5 \qquad n_2=960$$

although the value of $\frac{a^2A^2}{g^4}=\frac{1}{5000}$ only, the coefficient $\left(3-4\frac{l^2}{a^2}\right)$ $=141$, (the length of the coil is great compared with its radius) so that the term in $\frac{a^2A^2}{g^4}$ is -0.0282, and investigation of the complete formula shows that the succeeding terms are -0.0127 and -0.0048, so that their neglect will give an error of over 1.5 per cent. (For precision calculations see Bull., Bureau of Standards, 8, pp. 61–64, 1912, for the complete formula.) (Sci. Paper No. 169.)

CONCENTRIC COAXIAL SOLENOIDS (OUTER COIL THE SHORTER)

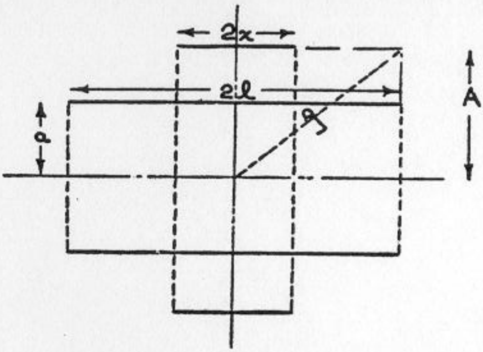

FIG. 206.—*Coaxial concentric solenoids, outer coil being shorter*

In this case we have to put $g=\sqrt{l^2+A^2}$, and the formula is

$$M=0.01974\frac{a^2n_1n_2}{g}\left[1+\frac{A^2a^2}{8g^4}\left(3-4\frac{x^2}{a^2}\right)\right] \tag{193}$$

which is rapidly convergent in most cases.

70. TABLES FOR INDUCTANCE CALCULATIONS

TABLE 8.—Values of δ in Formulas (132), (134), (137), (138), (140), (141), (142), and (147), for Calculating Inductance of Straight Wires at Any Frequency

x	δ	x	δ
0	0.250	12.0	0.059
0.5	.250	14.0	.050
1.0	.249	16.0	.044
1.5	.247	18.0	.039
2.0	.240	20.0	.035
2.5	0.228	25.0	0.028
3.0	.211	30.0	.024
3.5	.191	40.0	.0175
4.0	.1715	50.0	.014
4.5	.154	60.0	.012
5.0	0.139	70.0	0.010
6.0	.116	80.0	.009
7.0	.100	90.0	.008
8.0	.088	100.0	.007
9.0	.078	∞	.000
10.0	.070		

TABLE 9.—Constants P and Q in Formulas (141), (142), (144), and (145)

$\frac{2h}{l}$	P	$\frac{l}{2h}$	Q	$\frac{2h}{l}$	P	$\frac{l}{2h}$	Q
0	0	0	1.0000	0.6	0.5136	0.6	1.2918
0.1	0.0975	0.1	1.0499	.7	.5840	.7	1.3373
.2	.1900	.2	1.0997	.8	.6507	.8	1.3819
.3	.2778	.3	1.1489	.9	.7139	.9	1.4251
.4	.3608	.4	1.1975	1.0	.7740	1.0	1.4672
.5	.4393	.5	1.2452				

TABLE 10.—Values of K for Use in Formula (153)

Diameter/Length	K	Difference	Diameter/Length	K	Difference	Diameter/Length	K	Difference
0.00	1.0000	−0.0209	2.00	0.5255	−0.0118	7.00	0.2584	−0.0047
.05	.9791	203	2.10	.5137	112	7.20	.2537	45
.10	.9588	197	2.20	.5025	107	7.40	.2491	43
.15	.9391	190	2.30	.4918	102	7.60	.2448	42
.20	.9201	185	2.40	.4816	97	7.80	.2406	40
0.25	0.9016	−0.0178	2.50	0.4719	−0.0093	8.00	0.2366	−0.0094
.30	.8838	173	2.60	.4626	89	8.50	.2272	86
.35	.8665	167	2.70	.4537	85	9.00	.2185	79
.40	.8499	162	2.80	.4452	82	9.50	.2106	73
.45	.8337	156	2.90	.4370	78	10.00	.2033	....
0.50	0.8181	−0.0150	3.00	0.4292	−0.0075	10.0	0.2033	−0.0133
.55	.8031	146	3.10	.4217	72	11.0	.1903	113
.60	.7885	140	3.20	.4145	70	12.0	.1790	98
.65	.7745	136	3.30	.4075	67	13.0	.1692	87
.70	.7609	131	3.40	.4008	64	14.0	.1605	78
0.75	0.7478	−0.0127	3.50	0.3944	−0.0062	15.0	0.1527	−0.0070
.80	.7351	123	3.60	.3882	60	16.0	.1457	63
.85	.7228	118	3.70	.3822	58	17.0	.1394	58
.90	.7110	115	3.80	.3764	56	18.0	.1336	52
.95	.6995	111	3.90	.3708	54	19.0	.1284	48
1.00	0.6884	−0.0107	4.00	0.3654	−0.0052	20.0	0.1236	−0.0085
1.05	.6777	104	4.10	.3602	51	22.0	.1151	73
1.10	.6673	100	4.20	.3551	49	24.0	.1078	63
1.15	.6573	98	4.30	.3502	47	26.0	.1015	56
1.20	.6475	94	4.40	.3455	46	28.0	.0959	49
1.25	0.6381	−0.0091	4.50	0.3409	−0.0045	30.0	0.0910	−0.0102
1.30	.6290	89	4.60	.3364	43	35.0	.0808	80
1.35	.6201	86	4.70	.3321	42	40.0	.0728	64
1.40	.6115	84	4.80	.3279	41	45.0	.0664	53
1.45	.6031	81	4.90	.3238	40	50.0	.0611	43
1.50	0.5950	−0.0079	5.00	0.3198	−0.0076	60.0	0.0528	−0.0061
1.55	.5871	76	5.20	.3122	72	70.0	.0467	48
1.60	.5795	74	5.40	.3050	69	80.0	.0419	38
1.65	.5721	72	5.60	.2981	65	90.0	.0381	31
1.70	.5649	70	5.80	.2916	62	100.0	.0350	....
1.75	0.5579	−0.0068	6.00	0.2854	−0.0059			
1.80	.5511	67	6.20	.2795	56			
1.85	.5444	65	6.40	.2739	54			
1.90	.5379	63	6.60	.2685	52			
1.95	.5316	61	6.80	2633	49			

TABLE 11.—Values of Correction Term A in Formulas (155), (165), (168), and (169)

$\frac{d}{D}$	A	Difference	$\frac{d}{D}$	A	Difference	$\frac{d}{D}$	A	Difference
1.00	0.557	−0.051	0.40	−0.359	−0.052	0.15	−1.340	−0.069
0.95	.506	54	.38	.411	54	.14	1.409	74
.90	.452	57	.36	.465	57	.13	1.483	80
.85	.394	61	.34	.522	61	.12	1.563	87
.80	.334	65	.32	.583	64	.11	1.650	96
0.75	0.269	−0.069	0.30	−0.647	−0.069	0.10	−1.746	−0.105
.70	.200	74	.28	.716	74	.09	1.851	.118
.65	.126	80	.26	.790	80	.08	1.969	.133
.60	.046	87	.24	.870	87	.07	2.102	.154
.55	− .041	95	.22	.957	96	.06	2.256	.173
0.50	−0.136	−0.041	0.20	−1.053	−0.051	0.05	−2.439	−0.223
.48	.177	43	.19	1.104	54	.04	2.662	.288
.46	.220	44	.18	1.158	57	.03	2.950	.405
.44	.264	47	.17	1.215	61	.02	3.355	.693
.42	.311	48	.16	1.276	64	.01	4.048	....

TABLE 12.—Values of Correction B in Formulas (155), (165), (168), and (169)

Number of turns, n	B	Number of turns, n	B
1	0.000	40	0.315
2	.114	45	.317
3	.166	50	.319
4	.197	60	.322
5	.218	70	.324
6	0.233	80	0.326
7	.244	90	.327
8	.253	100	.328
9	.260	150	.331
10	.266	200	.333
15	0.286	300	0.334
20	.297	400	.335
25	.304	500	.336
30	.308	700	.336
35	.312	1000	.336

TABLE 13.—Values of B_s for Use in Formula (156)

$\frac{b}{c}$	B_s	$\frac{b}{c}$	B_s
1	0.0000	16	0.3017
2	.1202	17	.3041
3	.1753	18	.3062
4	.2076	19	.3082
5	.2292	20	.3099
6	0.2446	21	0.3116
7	.2563	22	.3131
8	.2656	23	.3145
9	.2730	24	.3157
10	.2792	25	.3169
11	0.2844	26	0.3180
12	.2888	27	.3190
13	.2927	28	.3200
14	.2961	29	.3209
15	.2991	30	.3218

TABLE 14.—Constants Used in Formulas (157) and (158)

b/c or c/b	y_1	Difference	c/b	y_2	Difference	b/c	y_3	Difference
0	0.5000	0.0253	0	0.125	0.002	0	0.597	0.002
0.025	.5253	237						
.05	.5490	434	0.05	.127	5	0.05	.599	3
.10	.5924	386	.10	.132	10	.10	.602	6
0.15	0.6310	0.0342	0.15	0.142	0.013	0.15	0.608	0.007
.20	.6652	301	.20	.155	16	.20	.615	9
.25	.6953	266	.25	.171	20	.25	.624	9
.30	.7217	230	.30	.192	23	.30	.633	10
0.35	0.7447	0.0198	0.35	0.215	0.027	0.35	0.643	0.011
.40	.7645	171	.40	.242	31	.40	.654	11
.45	.7816	144	.45	.273	34	.45	.665	12
.50	.7960	121	.50	.307	37	.50	.677	13
0.55	0.8081	0.0101	0.55	0.344	0.040	0.55	0.690	0.012
.60	.8182	83	.60	.384	43	.60	.702	13
.65	.8265	66	.65	.427	47	.65	.715	14
.70	.8331	52	.70	.474	49	.70	.729	13
0.75	0.8383	0.0039	0.75	0.523	0.053	0.75	0.742	0.014
.80	.8422	29	.80	.576	56	.80	.756	15
.85	.8451	19	.85	.632	59	.85	.771	15
.90	.8470	10	.90	.690	62	.90	.786	15
0.95	0.8480	0.0003	0.95	0.752	0.064	0.95	0.801	0.015
1.00	.8483	....	1.00	.816	....	1.00	.816	....

TABLE 15.—Values of Constants in Formula (162)

ν	Values of δ_{12}					
	$\tau=0$	0.1	0.3	0.5	0.7	0.9
0	0.114	0.113	0.106	0.092	0.068	0.030
0.5	.090	.089	.083	.070	.049	.020
1.0	.064	.064	.059	.050	.034	.013
1.5	.047	.046	.043	.036	.025	.009
2.0	.035	.035	.032	.027	.018	.007
3.0	.022	.022	.020	.017	.011	.004
4.0	.015	.015	.014	.012	.008	.003
6.0	.008	.008	.008	.006	.004	.002
8.0	.006	.006	.005	.004	.003	.001
10.0	.004	.004	.004	.003	.002	.001

ν	Values of δ_{13}			
	$\tau=0$	0.3	0.6	0.9
0	0.022	0.020	0.014	0.004
0.5	.021	.019	.014	.004
1.0	.019	.018	.013	.004
2.0	.015	.015	.010	.003
4.0	.008	.008	.005	.002
6.0	.005	.005	.004	.001
10.0	.003	.003	.002	.005

ν	Values of δ_{14}			
	$\tau=0$	0.3	0.6	0.9
0	0.009	0.009	0.006	0.002
1	.009	.008	.006	.002
3	.007	.006	.004	.001
5	.004	.004	.003	.001
10	.002	.002	.001	.000

ν	Values of δ_{15}			
	$\tau=0$	0.1	0.5	0.9
0	0.005	0.005	0.004	0.001
5	.003	.003	.002	.001
10	.002	.002	.001	.000

ν	Values of δ_{16}		
	$\tau=0$ and 0.1	0.5	0.9
0	0.003	0.003	0.001
5	.002	.002	.000
10	.001	.001	.000

ν	Values of δ_{17}		
	$\tau=0$ and 0.1	0.5	0.9
0	0.002	0.002	0.001
5	.002	.001	.000
10	.001	.001	.000

ν	Values of δ_{18}		
	$\tau=0$ and 0.1	0.5	0.9
0	0.002	0.001	0.000
5	.001	.001	.000
10	.001	.001	.000

NOTE.—The maximum values of all further values of the δ's are 0.001 or less.

TABLE 16.—Values of F in Formula (187) for the Calculation of the Mutual Inductance of Coaxial Circles

r_2/r_1	F	Difference
0	∞	
0.010	0.05016	−0.00120
.011	4897	109
.012	4787	100
0.013	4687	−0.00093
.014	4594	87
.015	4507	81
.016	4426	148
.018	4278	132
0.020	0.04146	−0.00119
.022	4027	109
.024	3918	100
.026	3818	93
.028	3725	86
0.030	3639	−0.00081
.032	3558	76
.034	3482	71
.036	3411	68
.038	3343	64
0.040	0.03279	−0.00061
.042	3218	58
.044	3160	55
.046	3105	53
.048	3052	51
0.050	0.03001	−0.00226
.060	2775	191
.070	2584	164
.080	2420	144
.090	2276	128
0.100	0.02148	−0.00116
.11	2032	104
.12	1928	96
.13	1832	89
.14	1743	82
0.15	0.01661	−0.00075
.16	1586	71
.17	1515	66
.18	1449	62
.19	1387	59
0.20	0.01328	−0.00055
.21	1273	52
.22	1221	50
.23	1171	47
.24	1124	45
0.25	0.010792	−0.000425
.26	10366	408
.27	0.009958	388
.28	9570	371
.29	9199	355

r_2/r_1	F	Difference
0.30	0.008844	−0.000341
.31	8503	328
.32	8175	314
.33	7861	302
.34	7559	290
0.35	0.007269	−0.000280
.36	6989	270
.37	6720	260
.38	6460	249
.39	6211	241
0.40	0.005970	−0.000232
.41	5738	225
.42	5514	217
.43	5297	210
.44	5087	202
0.45	0.004885	−0.000195
.46	4690	189
.47	4501	183
.48	4318	178
.49	4140	171
0.50	0.003969	−0.000166
.51	3803	160
.52	3643	156
.53	3487	150
.54	3337	146
0.55	0.003191	−0.000141
.56	3050	137
.57	2913	133
.58	2780	128
.59	2652	125
0.60	0.002527	−0.000120
.61	2407	117
.62	2290	113
.63	2177	109
.64	2068	106
0.65	0.001962	−0.000103
.66	1859	99
.67	1760	96
.68	1664	93
.69	1571	90
0.70	0.001481	−0.000087
.71	1394	84
.72	1310	81
.73	1228	78
.74	1150	76
0.75	0.0010741	−0.0000731
.76	10010	704
.77	9306	680
.78	8626	653
.79	7973	628

r_2/r_1	F	Difference
0.80	0.0007345	−0.0000604
.81	6741	579
.82	6162	555
.83	5607	531
.84	5076	507
0.85	0.0004569	−0.0000484
.86	4085	460
.87	3625	437
.88	3188	413
.89	2775	389
0.90	0.0002386	−0.0000365
.91	2021	341
.92	1680	316
.93	1364	290
.94	1074	263
0.95	0.00008107	−0.00002351
.96	5756	2046
.97	3710	1706
.98	2004	1301
.99	703	703
1.00	0	....
0.950	0.00008107	−0.00000494
.952	7613	482
.954	7131	470
.956	6661	458
.958	6202	446
0.960	0.00005756	−0.00000436
.962	5320	421
.964	4899	409
.966	4490	397
.968	4093	383
0.970	0.00003710	−0.00000370
.972	3340	356
.974	2984	341
.976	2643	327
.978	2316	312
0.980	0.00002004	−0.00000296
.982	1708	278
.984	1430	262
.986	1168	242
.988	926	223
0.990	0.00000703	−0.00000201
.992	502	177
.994	326	148
.996	177	115
.998	062	62

DESIGN OF INDUCTANCE COILS

71. DESIGN OF SINGLE-LAYER COILS

The problems of design of single-layer coils may be broadly classified as of two kinds.

(1) Where it is required to design a coil which shall have a certain desired inductance with a given length of wire, the choice of dimensions of the winding and kind of wire to be used being unrestricted within rather broad limits. This class of problems of design includes a consideration of the question as to what

shape of coil will give the required inductance with the minimum resistance.

(2) Given a certain winding form or frame, what pitch of winding and number of turns will be necessary, if a certain inductance is to be obtained.

In the following treatment of the problem the inductance of the coil will be assumed as equal to that of the equivalent cylindrical current sheet. This is allowable, since, in general, the correction for the cross section of the wire will not amount to more than 1 per cent of the total inductance, an amount which may be safely neglected in making the design. The formulas to be given may, of course, be used for making a calculation of the inductance of a given coil. Nevertheless, since their practical use is made to depend upon the interpolation of numerical values from a graph, for accurate calculations formulas (153) and (155) should be used.

The inductances of coils of different size, but of identical shape, and the same number of turns, are proportional to the ratio of their linear dimensions. Every formula for the inductance should, accordingly, be capable of expression in terms of some single chosen linear dimension, all the other dimensions occurring in the formula in pairs in the form of ratios.

Two formulas are here developed, the first applicable to the solution of problems of the first class, giving the inductance in terms of the total length of wire l, the second for problems presupposing a winding frame of given dimensions. Both show the dependence of the inductance on the shape of the coil.

Coil of Minimum Resistance.—The fundamental relations of the constants of a coil are

$$l = 2\pi a n \qquad b = nD$$

$$L_s = 4\pi^2 n^2 \frac{a^2}{b} K \text{ cgs units}$$

the constant K being a function of the shape factor $\frac{2a}{b}$, diameter $\div$ length (Table 10, p. 283).

The expression for the inductance may be written as

$$L_s = \frac{2\pi a l n}{b} K$$

and n may be eliminated by substituting for it the expression

$$n = \sqrt{\frac{lb}{2\pi a D}} = \nu \sqrt{\frac{l}{D}},$$

obtained by multiplying together the two expressions involving n above. There results, then,

$$L_s = l\sqrt{\pi \frac{2a}{b} \frac{l}{D}} \cdot K \quad \text{cgs units}$$

or

$$L_s = \frac{l^{\frac{3}{2}}}{\sqrt{D}} \frac{K}{1000} \sqrt{\pi \frac{2a}{b}} = \frac{l^{\frac{3}{2}}}{\sqrt{D}} F \text{ microhenries.} \qquad (194)$$

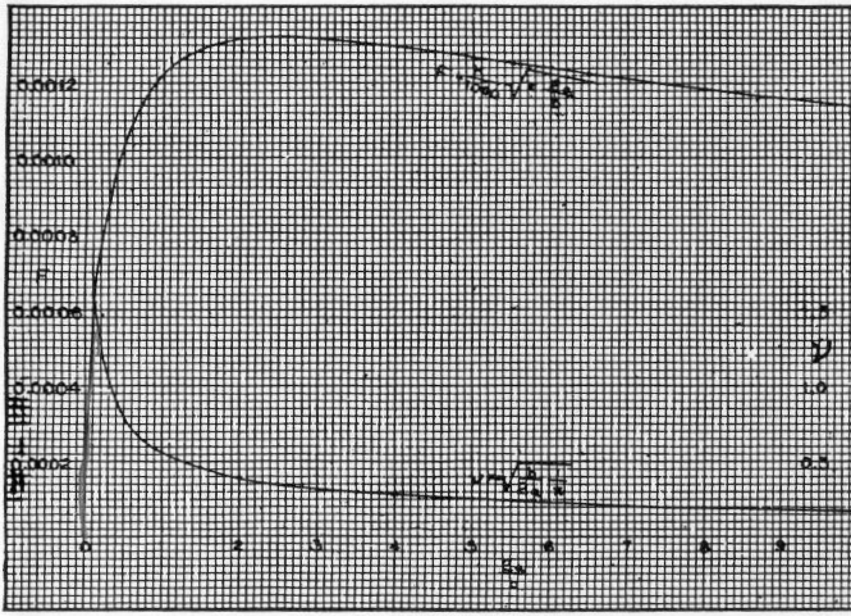

FIG. 207.—*(1) Variation of F with different ratios of coil diameter to length; (2) variations of ν with ratios of diameter to length*

To aid in the use of this formula the curve of Fig. 207 has been prepared, which enables the value of $F = \frac{K}{1000}\sqrt{\pi \frac{2a}{b}}$ to be obtained for any desired value of $\frac{2a}{b}$. The formula (194) and the curve enable one to obtain with very little labor the approximate value of the inductance which may be obtained in a coil of given shape with given l and D. On the same figure is also plotted the factor $\nu = \sqrt{\frac{b}{\pi 2a}}$ as a function of $\frac{2a}{b}$ (see example below).

Coil Wound on Given Form.—To obtain the second formula, we substitute for n its value $\frac{b}{D}$, and

$$L_s = 4\pi^2 \frac{b^2}{D^2} \frac{a^2}{b} K = 2a\pi^2 \left(\frac{2a}{D}\right)^2 \frac{b}{2a} K \text{ cgs units}$$

or

$$L_s = \frac{(2a)^3}{D^2} \left[\frac{\pi^2}{1000} \frac{b}{2a} K \right] \text{microhenries} \tag{195}$$

and, finally,

$$\frac{(2a)^3}{L_s D^2} = f \tag{196}$$

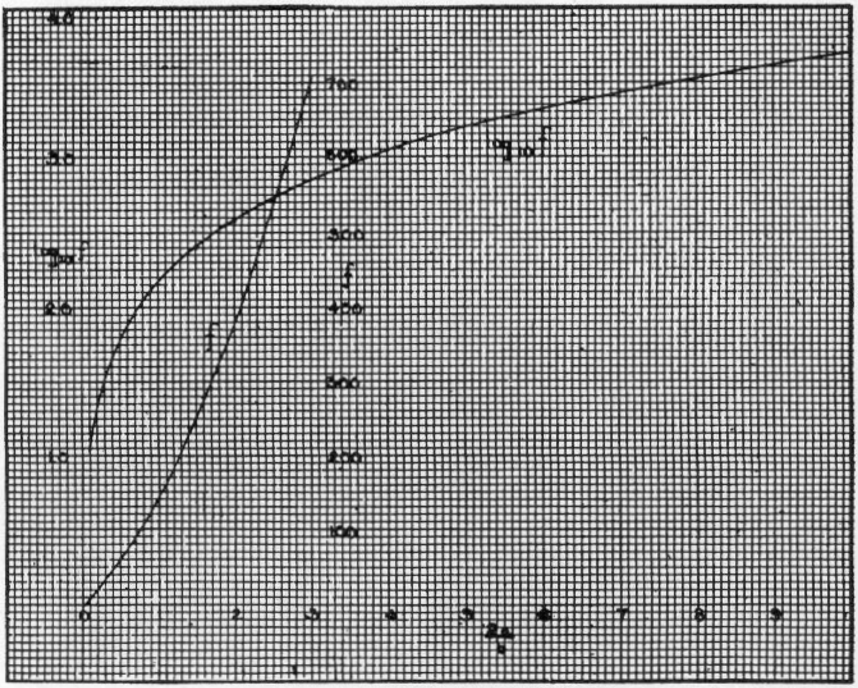

FIG. 208.—*Variation of f and $\log_{10} f$ with $\frac{2a}{b}$*

To aid in making calculations the curves of Fig. 208 have been prepared, which give the values of f and $\log_{10} f = \log_{10} \left[\frac{1000}{\pi^2 K} \frac{2a}{b} \right]$ for different values of $\frac{2a}{b}$. The value of $\log_{10} f$ is plotted, rather than that of f, for large values of $\frac{2a}{b}$, to enable values to be interpolated with greater accuracy.

From formula (194) and Fig. 207 it is at once evident that with a given length of wire, wound with a given pitch, that coil has the greatest inductance, which has such a shape that the ratio $\frac{\text{diameter}}{\text{length}}=2.46$ approximately. Or, to obtain a coil of a certain desired inductance, with a minimum resistance, this relation should be realized. However, although the inductance diminishes rather rapidly for longer coils than this, changes in the direction of making the coil shorter relative to the diameter are not important over rather wide limits. Naturally, other considerations may modify the design appreciably. These other considerations include the distributed capacity of the coil and the variation of resistance with frequency.

Example.—Given the pitch of winding, the shape of the coil $\left(\frac{2a}{b}\right)$, and the inductance, to determine the length of wire necessary, the dimensions of the coil and the number of turns.

Assuming $D=0.2$ cm, $\frac{2a}{b}=2.6$, $L_s=1000$ microhenries,

By formula (194), $l^{\frac{3}{2}}=\frac{1000\sqrt{0.2}}{0.001322}$, (the value of $F=0.001322$ being taken from the curve of Fig. 207) or $l=4850$ cm. The number of turns may be obtained immediately from the relation $n=\sqrt{\frac{l}{D}}\sqrt{\frac{b}{2\pi a}}=\nu\sqrt{\frac{l}{D}}$ and the graph of ν.

log 1000	$=3.$
½ log 0.2	$=\bar{1}.65052$
	2.65052
log F	$=\bar{3}.12123$
$^3/_2$ log l	$=5.52929$
⅓ log l	$=1.84310$
log l	$=3.68619$

Here $n=\sqrt{\frac{4850}{0.2}}\,(0.350)=54.5$ turns, and $b=nD=10.9$ cm, while $2a=2.6\times10.9=28.3$ cm.

If the pitch of the winding had been assumed greater, or a coil of much larger inductance were required, the design of the coil would call for larger dimensions, and cases may arise where the design may prove unsatisfactory, because the coil would be too large. The effect of changing the length and pitch, the shape being taken constant, may be seen from (194), which shows that $L_s\propto\frac{l^{\frac{3}{2}}}{\sqrt{D}}$, so that a given fractional increase in the length of the wire is more effective in increasing the inductance than the same fractional decrease in the pitch. The number of turns depends on $\sqrt{\frac{l}{D}}$ the shape of the coil being kept the same.

Example.—Formula (194) will also enable the question to be answered as to what pitch must be used if a given length of wire is to be wound with a certain shape of coil to give a desired inductance. If the pitch comes out smaller than the diameter of the proposed wire, the assumed length of wire must be increased.

Suppose that an inductance of 10 000 microhenries is desired with 50 meters of wire, the value of $\frac{2a}{b}$ being taken as 2.6, as before.

Then

$$\sqrt{D}=\frac{l^{\frac{3}{2}}}{L_s}F=\frac{(5000)^{\frac{3}{2}}\,0.001322}{10\ 000},\text{ or } D=0.00218\text{ cm},$$

which is manifestly impracticably small.

The maximum inductance attainable with the given length of wire could be found by solving (194) for L with the smallest practicable pitch substituted for D, that value being used for F, which corresponds to the assumed ratio of diameter to length.

Example.—Suppose we have a winding form of given diameter $2a=10$ cm, how many turns of wire will have to be used for an inductance of $1000\mu h$ if the winding pitch is taken as 0.2, and what will be the axial length of the winding?

From (196)

$$f=\frac{1000}{1000\times 0.04}=25 \text{ or } \log_{10}f=1.398$$

From Fig. 208 this corresponds to a value of $\frac{2a}{b}=0.225$, or b must be 45 cm, and the number of turns $n=\frac{b}{D}=\frac{45}{0.2}=225$. Such a coil would be too long to be convenient. A smaller pitch should be used.

Example.—Suppose we have given the same winding form, and we wish to find what pitch is necessary for an inductance of $1000\mu h$, in order that the length of the coil shall not be greater than the diameter.

For

$$\frac{2a}{b}=1,\ f=148\ \text{(Fig. 208)}$$

and by (196)

$$D^2=\frac{(2a)^3}{L_s f}=\frac{1000}{1000\times 148}\text{ or } D=0.082$$

This is a pretty close winding, showing that the winding form has rather too small a diameter for a coil of this inductance.

Example.—To find the diameter of a winding form to give an inductance of $1000 \mu h$, with a shape ratio $\frac{2a}{b} = 2.6$, the pitch being chosen as 0.2 cm.

From (196) we have $(2a)^3 = L_s D^2$.

The value of f for $\frac{2a}{b} = 2.6$ is (from Fig. 208) given by $\log_{10} f = 2.75$ or $f = 565$ approximately. Therefore $(2a)^3 = 1000 \times 0.04 \times 565$, or $2a = 28.2$ cm, which will give $b = 10.85$, $n = 54.2$.

If, instead, the shape is assumed to be given by $\frac{2a}{b} = 1$, then $\log f = 2.17$ or $f = 148$.

$$(2a)^3 = 1000 \times 0.04 \times 148, \text{ or } 2a = 18.1 \text{ cm} = b, \text{ and } n = 90.5.$$

The values of f taken from Fig. 208 are not so precise as could be calculated from the equation (195), but the accuracy should suffice for this kind of work.

72. DESIGN OF MULTIPLE-LAYER COILS

For purposes of design we may neglect the correction for cross section of the wire, formula (159), and operate on formulas (157) and (158) alone.

Two forms of equation have been found useful, the first involving the length of wire in the coil and the second the mean radius of the coil.

Suppose that the length of the winding l, the distance between the centers of adjacent wires D, shape of cross section $\frac{b}{c}$, and the shape ratio of the coil $\frac{c}{a}$, are given. We obtain an expression for n by multiplying together the fundamental equations,

$$n = \frac{bc}{D^2} = \frac{b}{c}\left(\frac{c}{D}\right)^2 \text{ and } n^2 = \frac{l^2}{(2\pi a)^2}$$

which involves ratios of known quantities only.

$$n = \left(\frac{l}{D}\right)^{\frac{2}{3}} \left(\frac{c}{a}\right)^{\frac{2}{3}} \left(\frac{b}{c}\right)^{\frac{1}{3}} \left(\frac{1}{2\pi}\right)^{\frac{2}{3}} \qquad (197)$$

In equation (158) the factor $4\pi a n^2 = 2ln$, and if the value of n just found, be introduced, we have finally for $c>b$

$$L=\sqrt[3]{\frac{2}{\pi^2}}\,\frac{l^{\frac{4}{3}}}{D^{\frac{2}{3}}}\left(\frac{c}{a}\right)^{\frac{2}{3}}\left(\frac{b}{c}\right)^{\frac{1}{3}}\Bigg[\log_e 8-\log_e\frac{c}{a}-\frac{1}{2}\log_e\left(1+\frac{b^2}{c^2}\right)-y_1$$
$$+\frac{c^2}{16a^2}\left\{y_2+\frac{1}{6}\left(1+3\,\frac{b^2}{c^2}\right)\left[\log_e\frac{8a}{c}-\frac{1}{2}\log_e\left(1+\frac{b^2}{c^2}\right)\right]\right\}\Bigg] \qquad (198)$$

and for $b>c$

$$L=\sqrt[3]{\frac{2}{\pi^2}}\,\frac{l^{\frac{4}{3}}}{D^{\frac{2}{3}}}\left(\frac{c}{a}\right)^{\frac{2}{3}}\left(\frac{b}{c}\right)^{\frac{1}{3}}\Bigg[\log_e 8-\log_e\frac{c}{a}-\log_e\frac{b}{c}-\frac{1}{2}\log_e\left(1+\frac{c^2}{b^2}\right)-y_1$$
$$+\frac{c^2}{16a^2}\,\frac{b^2}{c^2}\left\{y_2+\frac{1}{2}\left(1+\frac{c^2}{3b^2}\right)\left[\log_e\frac{8a}{c}-\log_e\frac{b}{c}-\frac{1}{2}\log_e\left(1+\frac{c^2}{b^2}\right)\right]\right\}\Bigg] \qquad (199)$$

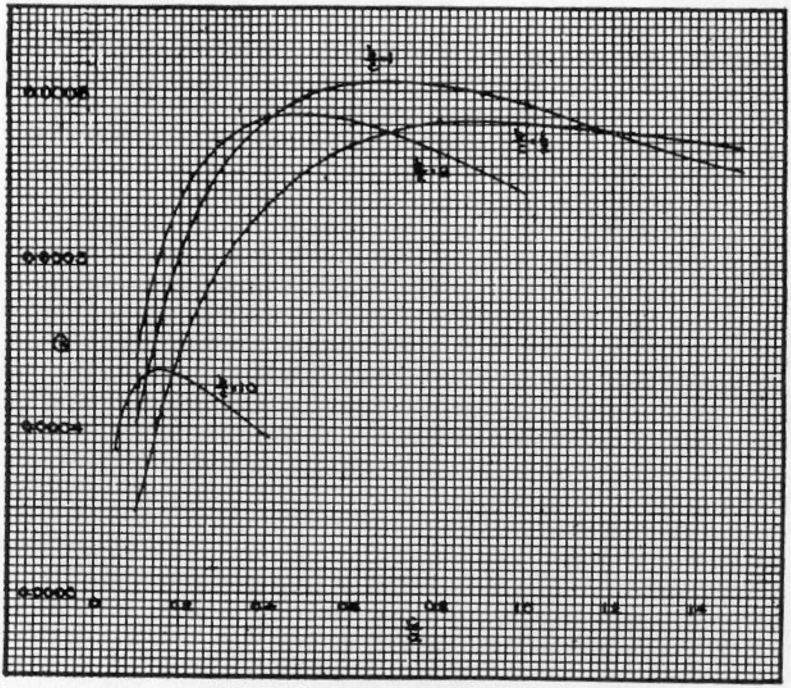

FIG. 209.—*Values of (G) for given values of $\frac{c}{a}$ and $\frac{b}{c}$*

Both of these equations may be written in the form

$$L=\frac{l^{\frac{4}{3}}}{D^{\frac{2}{3}}}\,G \text{ microhenries} \qquad (200)$$

in which G is a factor whose value for given values of $\frac{c}{a}$ and $\frac{b}{c}$ may be taken from the curves of Fig. 209.

When l is known

$$a = \sqrt[3]{\frac{l}{2\pi} \frac{c}{b} \frac{D^2}{(c/a)^2}} \qquad (201)$$

From these curves one can see that, for a square cross section, $b/c = 1$, the inductance of a given length of wire is a maximum for a value of $\frac{c}{a}$ equal to about $\frac{2}{3}$. Investigation shows that this point is, more exactly, $c/a = 0.662$; that is, for a mean diameter of coil = 3.02 times the side of the cross section. Further, for a given resistance and shape of coil, the square cross section gives a greater inductance than any other form.

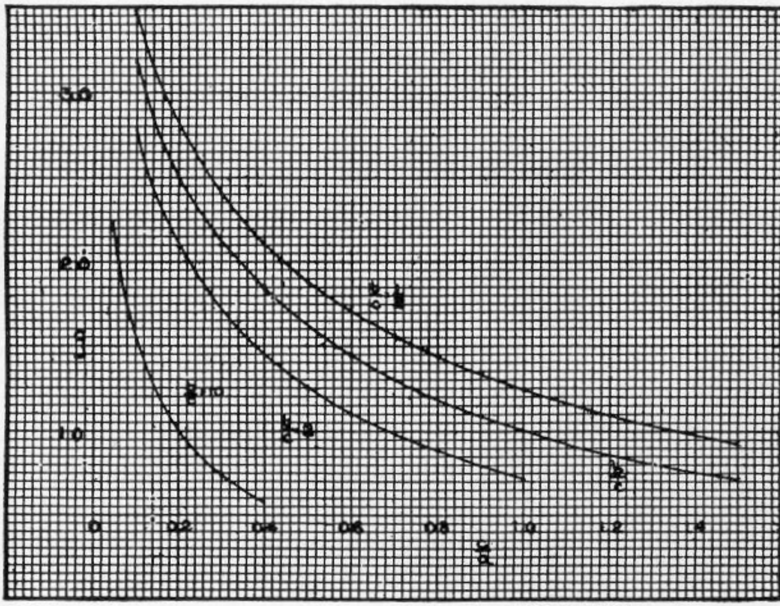

FIG. 210.—*Values of* (g) *for given values of* $\frac{c}{a}$ *and* $\frac{b}{c}$

The second design formula supposes that the dimensions a, c, and $\frac{b}{c}$ of the winding form are given, together with the pitch of the winding. The expressions (157) and (158) for the inductance may then be written

$$L = 0.01257\, a \frac{b^2}{c^2} \left(\frac{c}{D}\right)^4 g \text{ microhenries} \qquad (202)$$

$$= 0.01257\, an^2 g \qquad (203)$$

The curves of Fig. 210, which give g for different values of $\frac{c}{a}$ and $\frac{b}{c}$ allow of interpolation of the proper value in any given case.

Example.—Suppose we have a wire of such a size that it may be wound 20 turns to the centimeter, and we wish to design a coil to have an inductance of 10 millihenries, to have a square cross section and such a mean radius as to obtain the desired inductance with the smallest resistance (smallest length of the wire).

The latter condition requires that $\frac{c}{a}=0.662$. The given quantities are $D=0.05$ cm, $b/c=1$. From Fig. 209 we find that $G=0.000606$, so that (200) becomes $10\,000=\frac{l^{5/3}}{(0.05)^{2/3}}\,0.000606$, from which $l=6458$ cm or 64.58 meters of wire.

$$
\begin{aligned}
2/3 \log D &= \bar{1}.13265\\
\log \frac{10^7}{0.606} &= 7.21753\\
5/3 \log l &= 6.35018\\
1/3 \log l &= 1.27004\\
2 \log l &= 7.62022\\
\log l &= 3.81011
\end{aligned}
$$

From the fundamental equation (201)

$$a=\sqrt[3]{\frac{l}{2\pi}\cdot\frac{c}{b}\cdot\frac{D^2}{(c/a)^2}}$$

$$=1.80$$

and thence $b=c=0.662\times1.80=1.19$, and $n=\frac{bc}{D^2}=\frac{(1.19)^2}{0.0025}=570$.

This coil is rather too small to allow of its dimensions being accurately measured.

If wire of double the pitch is used, the design works out with the following results

$$
\begin{aligned}
l &= 85.22 \text{ meters} \qquad & c=b&=2.08\\
n &= 432 & a&=3.18
\end{aligned}
$$

which is more suitable.

Example.—We have a form whose dimensions are $2a=10$, $c=3$, $b=2.4$, wound with wire of such a size that there are 10 turns per cm; that is, $D=0.1$. What is the inductance obtained and what length of wire is used?

$$n=\frac{bc}{D^2}=\frac{3\times2.4}{0.01}=720$$

From Fig. 210 the interpolated value of g for $\frac{b}{c}=0.8$, $c/a=0.6$ is 1.54 (calculated directly from (158) = 1.552). Accordingly,

$$L=0.01257\times5\times\overline{720}^2\times1.54=50\ 160\ \mu h.$$
$$=50.16 \text{ millihenries.}$$

The length of wire is $l=2\pi\ an=10\ \pi\ 720=22\ 600$ cm
$=226$ meters.

Example.—The same formula might be used to answer the question, How many turns would have to be wound (completely filling this cross section) in order to obtain a desired inductance, say 20 millihenries. From (203),

$$n^2=\frac{L}{0.01257\ ag}=\frac{20\ 000}{(0.01257)\ 5\ (1.54)}=206\ 500$$

or n would be 454, which would mean that

$$D^2=\frac{bc}{454}=\frac{7.20}{454}=0.0158$$

or $D=0.126$, so that the wire would have to wind about 8 turns to the centimeter.

The skin effect and capacity between the layers of the wire are larger in this kind of coil than in the other forms previously considered. A multiple layer coil is therefore to be regarded as undesirable in radio work, and if it be used the cross section should be made small relative to the mean radius.

73. DESIGN OF FLAT SPIRALS

The design of a flat spiral differs from that of a multiple layer coil in that the actual width b of the tape used (not b/c) is supposed to be a given quantity.

The fundamental equations are

$$n=\frac{c}{D} \text{ and } n=\frac{l}{2\pi a},$$

which, on multiplication, give

$$n=\sqrt{\frac{1}{2\pi}\cdot\frac{c}{a}\cdot\frac{l}{D}} \qquad (204)$$

and this introduced into the expression $4\pi a n^2 = 2ln$ gives finally

$$L = \frac{l^{\frac{3}{2}}}{\sqrt{D}} \sqrt{\frac{2}{\pi}\frac{c}{a}} \Bigg[\left\{ \log_e 8 - \log_e \frac{c}{a} - \tfrac{1}{2} \log_e \left(1 + \frac{b^2}{c^2} \right) - y_1 \right\} + \frac{1}{16}\frac{c^2}{a^2} \left[y_2 + \frac{1}{6} \left(1 + \frac{3b^2}{c^2} \right) \left\{ \log_e \frac{8a}{c} - \tfrac{1}{2} \log_e \left(1 + \frac{b^2}{c^2} \right) \right\} \right] \Bigg]$$

$$= \frac{l^{\frac{3}{2}}}{\sqrt{D}} H \text{ microhenries.} \qquad (205)$$

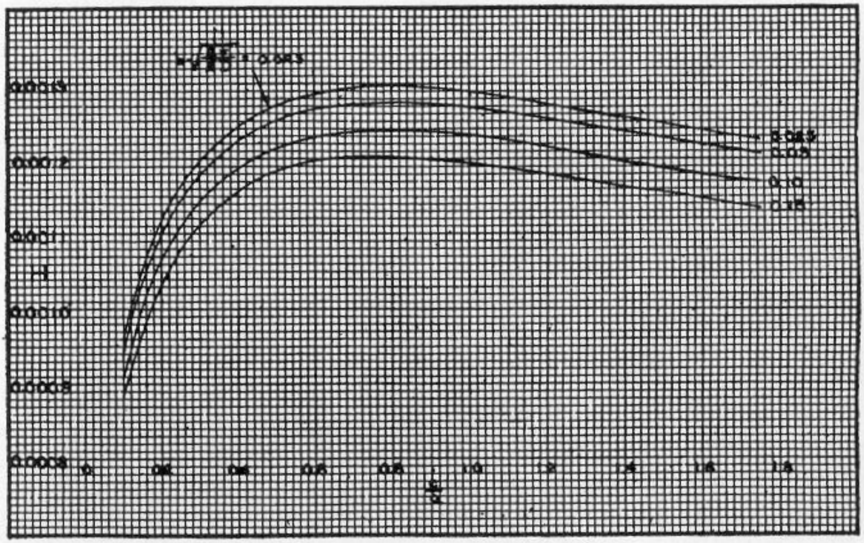

FIG. 211.—*Value of (H) for given values of* $\frac{c}{a}$ *and* $\frac{b}{c}$

The factor H, which may be determined from the curves of Fig. 211 is a function of c/a and b/c. The latter quantity may be expressed in terms of the known quantities by the equation

$$\frac{b}{c} = b\sqrt{\frac{2\pi}{lD}} \div \sqrt{\frac{c}{a}} \qquad (206)$$

Accordingly, the curves are plotted with H as ordinates, c/a as abscissas, and $b\sqrt{\frac{2\pi}{lD}}$ as parameter.

An important deduction which may be made from the curves is that for the maximum inductance with a given length of tape the ratio c/a should be about ¾, which means that the opening of the spiral should have a radius nearly as great as the dimension across

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the turns of the spiral. This point in design is in agreement with the practical observation that turns in the center of the spiral add a disproportionate amount to the high-frequency resistance of the spiral.

Example.—Find the length of tape 0.6 cm wide, wound with a pitch of 0.6 cm, to give an inductance of 200 μh, assuming such proportions that $c/a=1$. Work out the design.

Since l is not known, the parameter $b\sqrt{\frac{2\pi}{lD}}$ is not known. Assume a value of 0.1 for the latter. Then for the value $c/a=1$ the curve (Fig. 211) gives $H=0.00123$.

Thence $l^{\frac{3}{2}}=\frac{200\sqrt{0.6}}{0.00123}$ or $l=3287$ cm. With this value of l, the parameter is $0.6\sqrt{\frac{2\pi}{1972}}$ or 0.0339, to which the value $H=0.00128$ corresponds (with $\frac{c}{a}=1$). Repeating the calculation of l with this value of H, we find $l=3370$ cm as a second approximation. The next approximation gives a parameter of 0.0335 and the values of H and l are sensibly unchanged.

error see p.2

Using this parameter in (206), $\frac{b}{c}=0.0335$ or $c=\frac{0.6}{0.0335}=17.9$ and the value of $a=17.9$ likewise. The number of turns will be $n=\frac{17.9}{0.6}$ = about 30.

Example.—We have 17.50 meters of tape 1 cm wide, which we wind with a pitch of 0.5 cm, to such a shape that $c/a=0.8$.

Here $D=0.5$, $l=1750$ cm, $b=1$. The parameter is $\sqrt{\frac{2\pi}{875}}=0.0847$, to which, for $c/a=0.8$, $H=0.001248$ corresponds.

$$L=\frac{(1750)^{\frac{3}{2}}}{\sqrt{0.5}}0.001248=129.2\ \mu h$$

$$\frac{b}{c}=\frac{0.0847}{\sqrt{0.8}}=0.0947\text{, by equation (206)}$$

$$c=\frac{1}{0.0947}=10.56\text{ cm.}$$

$$a=\frac{10.56}{0.8}=13.2$$

and the number of turns, $n=\frac{10.56}{0.5}=21$ nearly.

Example.—The problem may arise as to how closely the tape in the preceding case would have to be wound, still keeping $\frac{c}{a}=0.8$, to obtain an inductance of 200 μh.

Changing the pitch D will change the parameter of the curves, and hence H. The changes in the latter will not be important, for small changes in D, so that to a first approximation the inductance will change inversely as $\sqrt{D}$.

Therefore

$$\sqrt{\frac{D}{0.5}}=\frac{129.2}{200}, \text{ or } D=0.2086 \text{ cm.}$$

Calculating the parameter with this value we find 0.1312, and thence $H=0.001216$, so that the second approximation is $\sqrt{D}=\frac{(1750)^{\frac{3}{2}}}{200}(0.001216)$, and $D=0.1981$, and another approximation is 0.197, the parameter being 0.1346. The dimensions are found from

$$\frac{b}{c}=\frac{0.1346}{\sqrt{0.8}}=0.1505 \qquad c=\frac{1}{0.1505}=6.64$$

$$a=\frac{c}{0.8}=8.30 \qquad n=\frac{6.649}{0.197}=34 \text{ nearly.}$$

HIGH-FREQUENCY RESISTANCE

74. RESISTANCE OF SIMPLE CONDUCTORS

Two principal causes act to increase the resistance of a circuit carrying a current of high frequency, above the value of its resistance with direct current, viz, the so-called skin effect and the capacity between the conductors. This section deals exclusively with the skin effect or change of resistance caused by change of current distribution within the conductor. (See sec. 3.)

Unfortunately, formulas for the skin effect are available only for the most simple circuits; and for other very common cases in practice only qualitative indications of the magnitude of the increase in resistance can be given.

In what follows

R = the resistance at frequency f

R_0 = the resistance with direct current or very low frequency alternating current.

The quantity of greatest practical interest is not R, but the resistance ratio $\frac{R}{R_0}$. Given this ratio for the desired frequency and the easily measured direct-current resistance, the high-frequency resistance follows at once.

The skin effect in a conductor always depends, in addition to

error see p.2

the thickness of the conductor, on the parameter $\sqrt{\frac{2\mu f}{\rho}}$, in which μ = permeability of the material, f = frequency of the current, ρ = the volume resistivity in microhm-cms, so that as far as skin effect is concerned, a thick wire at low frequencies may show as great a skin effect as a thin one at much higher frequency.

The skin effect is greater in good conductors than in wires of high resistivity, and conductors of magnetic material show an exaggerated increase of resistance with frequency.

Cylindrical Straight Wires.—For this case accurate values of the resistance ratio are given by the formula and tables here given.

If d is the diameter of the cross section of the wire in cm, the quantity

$$x = \pi d \sqrt{\frac{2\mu f}{\rho}} \sqrt{\frac{1}{1000}} \tag{207}$$

must be calculated (or, in the case of copper, obtained for the desired frequency from Table 19, p. 311 and formula (209)). Knowing the value of x, the value of $\frac{R}{R_0}$ may be taken at once from Table 17, page 309, which gives the value of $\frac{R}{R_0}$ directly for a wide range of values of x.

Table 19 gives values of

error see p.2

$$a_0 = 0.01071\sqrt{f} \tag{208}$$

for a copper wire at 20° C, 0.1 cm in diameter, and at various frequencies. The value of x for a copper wire of diameter d in cm is

$$x_0 = 10 d a_0 \tag{209}$$

For a material of resistivity ρ and permeability μ, the parameter x may also be simply obtained from the value which holds for a copper wire of the same diameter, by multiplying the latter value by $\sqrt{\mu\frac{\rho_0}{\rho}}$.

The range of Table 19 may be considerably extended by remembering that a is proportional to $\sqrt{f}$ or $\sqrt{\frac{1}{\lambda}}$, where λ is the wave length.

Table 18, page 310, will be found useful, when it is desired to determine what is the largest diameter of wire of a given material, which has a resistance ratio of not more than 1 per cent greater than unity. These values are, of course, based on certain assumed values of resistivity; temperature changes and differences of chemical composition will slightly alter the values. In the case of iron wires μ is the effective permeability over the cycle. This will, in general, be impossible to estimate closely. The values given show plainly how important is the skin effect in iron wires.

For a resistance ratio only one-tenth per cent greater than unity the values in Table 18 should be multiplied by 0.55, and for a 10 per cent increase of the high-frequency resistance the diameters given in the table must be multiplied by 1.78.

The formulas above given apply only to wires which are too far away from others to be affected by the latter. For wires near together, as, for example, in the case of parallel wires forming a return circuit, the mutual effect of one wire on the other always increases the ratio $\frac{R}{R_0}$. No formula for calculating this effect is available, but it is only for wires nearly in contact that it is important. At distances of 10 to 20 cm the mutual effect is entirely negligible.

Tubular Conductors.—The resistance ratio of tubular conductors in which the thickness of the walls of the tube is small in comparison with the mean diameter of the tube, may be calculated by the theoretical formula for an infinite plane of twice the thickness of the walls of the tube.

The value of the resistance ratio for this case may be obtained directly from Table 20, page 311, in terms of the quantity

$$\beta = x\tau\sqrt{2}, \qquad (210)$$

error see p.2

where

τ = the thickness of the walls of the tube in cm
x = the parameter defined in formula (207).

For copper tubes the parameter β_0 may be obtained very simply from the values of a_0 in Table 19, page 311, and the relation $\beta_0 = 10\sqrt{2}\,\tau\, a_0$.

For values of β greater than 4 no table is necessary, since we have simply, with an accuracy always greater than one-tenth of 1 per cent,

$$\frac{R}{R_0}=\beta \qquad (211)$$

Sufficient experimental evidence is not available to indicate an accurate method of procedure in the case of tubing where the ratio of diameter to wall thickness is not large. Measurements with tubing in which this ratio is as small as two or three indicate that approximate values of $\frac{R}{R_0}$ for this case may be calculated by using for τ, in the calculation of the parameter β, a value equal to two-thirds of the actual thickness of the walls of the tube.

Tubing which is very thin in comparison with its radius has, for the same cross section, a smaller high-frequency resistance than any other single conductor. For this reason galvanized-iron pipe is a good form of conductor for some radio work, the current all flowing in the thin layer of zinc. A conductor of smaller resistance than a tube of a certain cross section is obtained by the use of very fine strands separated widely from one another; there are practical difficulties, however, in making the separation great enough.

In a return circuit of tubular conductors the distance between the conductors should be kept as great as 10 or 20 cm. For tubular conductors nearly in contact the resistance ratio may be double that for a spacing of a few centimeters.

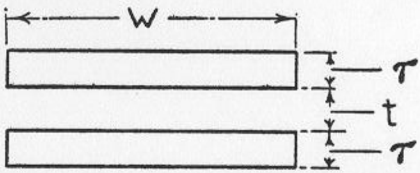

FIG. 212.—*Cross section of strip conductors forming a return circuit with narrow surfaces in the same plane*

Strip Conductors.—If two strips form together a return circuit and they are so placed that there is only a small thickness of dielectric between the wider face of one and the same face of the other (Fig. 212), the resistance ratio may be calculated by formula (210), using for τ the actual thickness of the strip.

As the thickness of the insulating space between the plates is increased, the accuracy of the formula decreases, but the error does not amount to more than a few per cent for values of this thickness as great as several centimeters.

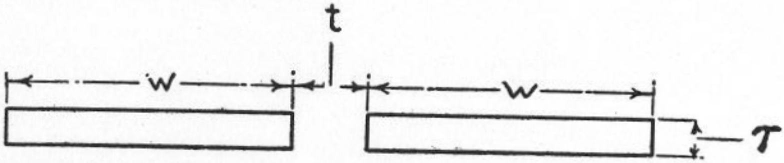

FIG. 213.—*Cross section of strip conductors forming a return circuit with wide surfaces in the same plane*

For a return circuit of strips placed with their wider faces in the same plane (Fig. 213), no formula is available. This is an unfavorable arrangement. As the distance t is reduced below a few centimeters the ratio $\frac{R}{R_o}$ increases rapidly and with the strips very close together may be as great as twice the value for the arrangement of Fig. 212.

For single strips—that is, for return circuits in which the distance between the conductors is so great that there is no appreciable mutual effect between the conductors—formula (210) is inapplicable owing to "edge effect"—the effect of the magnetic field produced by the current in the center of the strip upon the outer portions of the cross section.

Thus the resistance ratio $\frac{R}{R_o}$ is greater in a wide strip than in a narrow one of the same thickness, and in every case the resistance ratio is greater than for the two juxtaposed strips of Fig. 212. For $\frac{R}{R_o}$ between 1 and 1.5, the increase over formula (210) is usually not greater than 10 per cent.

Strips of square, or nearly square, cross section have values of $\frac{R}{R_o}$ not very different from those which hold for round conductors of the same area of cross section, the values being greater for the square strip than for the round conductor whose diameter is equal to the side of the square.

Simple Circuits of Round or Rectangular Wire.—The ratio of the resistance at high frequencies to that with direct current may be accurately obtained from Table 17, page 309, for circles or rectangles of round wire and in fact for any circuit of which the length is great compared with the thickness of the wire, provided no considerable portions of the circuit are placed close together. In the latter case, the resistance ratio is somewhat increased beyond the value calculated by the previous method and by an amount which can not be calculated.

The resistance ratio for a circuit of wire of rectangular section may be treated by the same method as for a single strip. If portions of the circuit are in close proximity, the precautions mentioned for two strips near together (p. 303) should be borne in mind.

75. RESISTANCE OF COILS

Single-Layer Coil; Wire of Rectangular Cross Section.—The only case for which an exact formula is available is that of a single-layer winding of wire of rectangular cross section with an insulation of negligible thickness between the turns, the length of the winding being assumed to be very great compared with the mean radius, and the latter being assumed very great compared with the thickness of the wire.

If R = the resistance at high frequency
R_0 = the resistance to direct current
τ = the radial thickness of the wire
b = the axial thickness of the wire
ρ = the volume resistivity of the wire in microhm-cm
ρ_0 = the volume resistivity of copper
μ = the permeability of the wire
D = the pitch of the winding,

then $\frac{R}{R_0}$ may be obtained directly from Table 20, page 311, having calculated first the quantity $\beta = 10\tau\sqrt{2}\,\alpha$, in which $\alpha = 0.1985\sqrt{\frac{\mu f}{\rho}}$. Values of α_0 for copper are given in Table 19, page 311, and the value of α for any other material is obtained from α_0 by the relation $\alpha = \alpha_0\sqrt{\mu\frac{\rho_0}{\rho}}$. For values of β greater than are included in Table 20 we have simply $\frac{R}{R_0} = \beta$.

In practice the ideal conditions presupposed above will not be realized. To reduce the value calculated for the idealized winding corrections need to be applied: (1) For the spacing of the wire, (2) for the round cross section of the wire, (3) for the curvature of the wire, (4) for the finite length of the coil.

Correction for Pitch of the Winding.—To take into account the fact that the pitch of the winding is not in general equal to the axial breadth of the wire an approximation is obtained if for β the argument

$$\beta' = \beta\sqrt{\frac{b}{D}} \text{ is substituted.}$$

For values of D greater than about $3b$ the values of $\frac{R}{R_0}$ thus obtained are too small.

Correction for the Round Cross Section of the Wire.—For coils of round wire only empirical expressions are known, and more experimental work is desirable.

To obtain an accuracy of perhaps 10 per cent in the resistance ratio the following procedure may be used:

Calculate first by (210) and Table 20, page 311, the resistance ratio $\frac{R'}{R_0'}$, supposing the coil to be wound with wire of square cross section of the same thickness as the actual diameter, taking into account the correction for the pitch of the winding. Then the resistance ratio $\frac{R}{R_0}$ for a winding of round wire will be found by the relation

$$\frac{R}{R_0} = 1 + 0.59\left[\frac{R' - R_0'}{R_0'}\right] \qquad (212)$$

Effect of Thickness of the Wire.—Although formula (210) holds only for a coil whose diameter is very great in comparison with the thickness of the wire, the error resulting from non-fulfillment of this condition will, in practical cases, be small compared with the other corrections and may be neglected.

Correction for Finite Length of the Coil.—For short coils the resistance ratio is greater than for long coils of the same wire, pitch, and radius, due to the appreciable strength of the magnetic field close to the wires on the outside of the coil.

No formulas are available for calculating this effect, but experiment seems to show that for short coils of thick wire at radio frequencies the resistance ratio may be expressed by

$$\frac{R}{R_0} = \frac{A}{\sqrt{\lambda}} + \frac{B}{\lambda^2} \qquad (213)$$

in which the first term represents the value as calculated by the formulas of the preceding section for long coils, while the constant of the second term has to be obtained by experiment. At long wave lengths the first term will predominate, but at very short wave lengths the second term may be equal or even larger than the first.

For round copper wires we may obtain the constant A by the relation $A = 15\,500\ dR_0$.

Multiple-Layer Coils.—For this case no accurate formulas have been derived. Experiment shows that the resistance ratio is much greater for a multiple-layer coil than for a single-layer coil of the same wire. Furthermore, the capacity of such a coil has, as already pointed out, a large effect on the resistance of the coil. Consequently, it is usually impossible to calculate even an approximate value for the change of resistance with frequency. At very high frequencies losses in the dielectric between the wires may cause an appreciable increase in the effective resistance of the coil. This effect is proportional to f^3.

76. STRANDED WIRE

The use of conductors consisting of a number of fine wires to reduce the skin effect is common. The resistance ratio for a stranded conductor is, however, always considerably larger than the value calculated by Table 19, page 311, and Table 17, page 309, for a single one of the strands. Only when the strands are at impracticably large distances from one another is this condition even approximately realized.

Formulas have been proposed for calculating the resistance ratio of stranded conductors,[85] but although they enable qualitatively correct conclusions to be drawn as to the effect of changing the frequency and some of the other variables, they do not give numerical values which agree at all closely with experiment. The cause for this lies, probably, to a large extent in the importance of small changes in the arrangement of the strands. The following general statements will serve as a rough guide as to what may be expected for the order of magnitude of the resistance ratio as an aid in design, but when a precise knowledge of the resistance ratio is required in any given case it should be measured. (See methods given in sections 46 to 50.)

Bare Strands in Contact.—The resistance ratio of n strands of bare wire placed parallel and making contact with one another is found by experiment to be the same as for a round solid wire

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[85] See references 112 to 123 of the Bibliography.

which has the same area of cross section as the sum of the cross-sectional areas of the strands; that is, n times the cross section of a single strand. This will be essentially the case in conductors that are in contact and are poorly insulated, except that at high frequencies the additional loss of energy due to heating of the imperfect contacts by the passage of the current from one strand to another may raise the resistance still higher.

Insulated Strands.—As the distance between the strands is increased, the resistance ratio falls, rapidly at first, and then more slowly toward the limit which holds for a single isolated strand. A very moderate thickness of insulation between the strands will quite materially reduce the resistance ratio, provided conduction in the dielectric is negligible.

Spiraling or twisting the strands has the effect of increasing the resistance ratio slightly, the distance between the strands being unchanged.

Transposition of the strands so that each takes up successively all possible positions in the cross section—as for example, by thorough braiding—reduces the resistance ratio but not as low as the value for a single strand.

Twisting together conductors, each of which is made up of a number of strands twisted together, the resulting composite conductor being twisted together with other similar composite conductors, etc., is a common method for transposing the strands in the cross section. Such conductors do not have a resistance ratio very much different from a simple bundle of well-insulated strands.

The most efficient method of transposition is to combine the strands in a hollow tube of basket weave. Such a conductor is naturally more costly than other forms of stranded conductor.

Effect of Number of Strands.—With respect to the choice of the number of strands, experiment shows that the absolute rise of the resistance in ohms depends on the diameter of a single strand, but is independent of the number of strands. Since, however, the direct-current resistance of the conductor is smaller the greater the number of the strands, the resistance ratio is greater the greater the number of strands. Reducing the diameter of the strands reduces the resistance ratio, the number of strands remaining unchanged, but to obtain a given current-carrying capacity, or a small enough total resistance, the total cross section must not be lowered below a certain limit, so that, in general, reducing the diameter of the strands means an increase in the number of strands.

With enameled strands of about 0.07 mm bare diameter twisted together to form a composite conductor the order of magnitude of the resistance ratio may be estimated by the following procedure. Calculate by Table 19, page 311, and Table 17, page 309, the resistance ratio for a single strand at the desired frequency (this value of R/R_o will lie very close to unity), and carry out the same calculation for the equivalent solid wire, whose diameter will of course be $d\sqrt{n}$, where n = the number of strands and d = the diameter of a single strand. Then the resistance ratio for the stranded conductor will, for moderate frequencies, lie about one-quarter to one-third of the way between these two values, being closer to the lower limit. This holds for straight wires up to higher frequencies than for solenoids. (See critical frequency mentioned in second paragraph below.) Not all so-called litzendraht is as good as this by any means. For a woven tube the resistance ratio may be as low as one-tenth of the way from the lower to the upper limits mentioned.

Coils of Stranded Wire.—In the case of solenoids wound with stranded conductor, the resistance ratio is always larger than for the straight conductor, and at high frequencies may be two to three times as great. It is appreciably greater for a very short coil than for a long solenoid.

For moderate frequencies the resistance ratio is less than for a similar coil of solid wire of the same cross section as just stated, but for every stranded-conductor coil there is a critical frequency above which the stranded conductor has the larger resistance ratio. This critical frequency lies higher the finer the strands and the smaller their number. For 100 strands of say 0.07 mm diameter this limit lies above the more usual radio frequencies.

This supposes that losses in the dielectric are not important, which is the case for single-layer coils with strands well insulated. In multiple-layer coils of stranded wire, dielectric losses are not negligible at high frequencies.

77. TABLES FOR RESISTANCE CALCULATIONS

TABLE 17.—Ratio of High-Frequency Resistance to the Direct-Current Resistance

[See formulas (207), (208), and (209)]

x	$\frac{R}{R_o}$	Difference
0	1.0000	0.0003
0.5	1.0003	.0004
.6	1.0007	.0005
.7	1.0012	.0009
.8	1.0021	.0013
.9	1.0034	.0018
1.0	1.005	0.003
1.1	1.008	.003
1.2	1.011	.004
1.3	1.015	.005
1.4	1.020	.006
1.5	1.026	.007
1.6	1.033	0.009
1.7	1.042	.010
1.8	1.052	.012
1.9	1.064	.014
2.0	1.078	.033
2.2	1.111	0.041
2.4	1.152	.049
2.6	1.201	.056
2.8	1.256	.062
3.0	1.318	.067
3.2	1.385	0.071
3.4	1.456	.073
3.6	1.529	.074
3.8	1.603	.075
4.0	1.678	.074
4.2	1.752	0.074
4.4	1.826	.073
4.6	1.899	.072
4.8	1.971	.072
5.0	2.043	.071

x	$\frac{R}{R_o}$	Difference
5.2	2.114	0.070
5.4	2.184	.070
5.6	2.254	.070
5.8	2.324	.070
6.0	2.394	.069
6.2	2.463	.070
6.4	2.533	0.070
6.6	2.603	.070
6.8	2.673	.070
7.0	2.743	.070
7.2	2.813	.071
7.4	2.884	.070
7.6	2.954	0.070
7.8	3.024	.070
8.0	3.094	.071
8.2	3.165	.070
8.4	3.235	.071
8.6	3.306	0.071
8.8	3.376	.070
9.0	3.446	.071
9.2	3.517	.070
9.4	3.587	.071
9.6	3.658	0.070
9.8	3.728	.071
10.0	3.799	.176
10.5	3.975	.176
11.0	4.151	.176
11.5	4.327	0.177
12.0	4.504	.176
12.5	4.680	.176
13.0	4.856	.177
13.5	5.033	.176

x	$\frac{R}{R_o}$	Difference
14.0	5.209	0.177
14.5	5.386	.176
15.0	5.562	.353
16.0	5.915	0.353
17.0	6.268	.353
18.0	6.621	.353
19.0	6.974	.354
20.0	7.328	.353
21.0	7.681	0.353
22.0	8.034	.353
23.0	8 387	.354
24.0	8.741	.353
25.0	9.094	.353
26.0	9.447	0.70
28.0	10.15	.71
30.0	10.86	.71
32.0	11.57	.70
34.0	12.27	.71
36.0	12.98	0.71
38.0	13.69	.71
40.0	14.40	.70
42.0	15.10	.71
44.0	15.81	.71
46.0	16.52	0.70
48.0	17.22	.71
50.0	17.93	3.54
60.0	21.47	3.53
70.0	25.00	3.54
80.0	28.54	3.53
90.0	32.07	3.54
100.0	35.61	....
∞	∞	

TABLE 18.—Maximum Diameter of Wires for High-Frequency Resistance Ratio of 1.01

Frequency + 10^6	0.1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	3.0
Wave length, meters	3000	1500	750	500	375	300	250	214.3	187.5	166.7	150	100
Material	Diameter in centimeters											
Copper	0.0356	0.0251	0.0177	0.0145	0.0125	0.0112	0.0102	0.0095	0.0089	0.0084	0.0079	0.0065
Silver	.0345	.0244	.0172	.0141	.0122	.0109	.0099	.0092	.0086	.0082	.0077	.0063
Gold	.0420	.0297	.0210	.0172	.0149	.0133	.0121	.0112	.0105	.0099	.0094	.0077
Platinum	.1120	.0793	.0560	.0457	.0396	.0354	.0323	.0300	.0280	.0264	.0250	.0205
Mercury	.264	.187	.132	.1080	.0936	.0836	.0763	.0706	.0661	.0623	.0591	.0483
Manganin	.1784	.1261	.0892	.0729	.0631	.0564	.0515	.0477	.0446	.0420	.0399	.0325
Constantan	.1892	.1337	.0946	.0772	.0664	.0598	.0546	.0506	.0473	.0446	.0423	.0345
German silver	.1942	.1372	0970	.0792	.0692	.0614	.0560	.0518	.0485	.0458	.0434	.0354
Graphite	.765	.541	.383	.312	.271	.242	.221	.204	.191	.180	.171	.140
Carbon	1.60	1.13	.801	.654	.566	.506	.462	.428	.400	.377	.358	.292
Iron $\mu=1000$	0.00263	0.00186	0.00131	0.00108	0.00094	0.00083	0.00076	.00070	0.00066	0.00062	0.00059	0.00048
$\mu=500$	.00373	.00264	.00187	.00152	.00132	.00118	.00108	.00100	.00093	.00088	.00084	.00068
$\mu=100$	.00838	.00590	.00418	.00340	.00295	.00264	.00241	.00223	.00209	.00197	.00186	.00152

TABLE 19.—Values of the Argument α_0 for Copper Wire 0.1 cm Diameter and Resistivity 1.724 Microhm-cms ($\alpha_0 = 0.01071\sqrt{f}$)

error
see p.2

f cycles per second	α_0	Difference	λ meters	f cycles per second	α_0	Difference	λ meters
100	0.1071	0.0443	....	50 000	2.395	0.229	6000
200	.1514	.0341	....	60 000	2.624	.210	5000
300	.1855	.0287	....	70 000	2.834	.195	4286
400	.2142	.0253	....	80 000	3.029	.184	3750
500	.2395	.0229	....	90 000	3.213	.174	3333
600	0.2624	0.0210	....	100 000	3.387	0.761	3000
700	.2834	.0195	....	150 000	4.148	.642	2000
800	.3029	.0184	....	200 000	4.790	.565	1500
900	.3213	.0174	....	250 000	5.355	.511	1200
1000	.3387	.1403	....	300 000	5.866	.318	1000
2000	0.4790	0.1076	....	333 333	6.184	0.380	900
3000	.5866	.0908	....	375 000	6.564	.452	800
4000	.6774	.0799	....	428 570	7.012	.561	700
5000	.7573	.0723	....	500 000	7.573	.723	600
				600 000	8.296	.664	500
6000	0.8296	0.0664	....				
7000	.8960	.0619	....	700 000	8.960	0.315	429
8000	.9579	.0581	....	750 000	9.275	.304	400
9000	1.0160	.055	....	800 000	9.579	.581	375
				900 000	10.16	.55	333
10 000	1.071	0.241	30 000	1 000 000	10.71	2.41	300
15 000	1.312	.202	20 000				
20 000	1.514	.341	15 000	1 500 000	13.12	5.43	200
30 000	1.855	.287	10 000	3 000 000	18.55	....	100
40 000	2.142	.253	7500				

TABLE 20.—Values of $\frac{R}{R_0}$ for Use with Formula (210)

β	$\frac{R}{R_0}$	Difference	β	$\frac{R}{R_0}$	Difference	β	$\frac{R}{R_0}$	Difference
0	1.000	....	1.0	1.086	0.037	2.5	2.477	0.111
0.1	1.000	....	1.1	1.123	.047	2.6	2.588	.109
.2	1.000	....	1.2	1.170	.059	2.7	2.697	.106
.3	1.001	....	1.3	1.229	.069	2.8	2.803	.104
.4	1.002	....	1.4	1.298	.080	2.9	2.907	.103
.5	1.006	0.002	1.5	1.378	.090	3.0	3.010	.101
0.55	1.008	.004	1.6	1.468	0.098	3.1	3.111	0.101
.60	1.012	.004	1.7	1.566	.106	3.2	3.212	.099
.65	1.016	.005	1.8	1.672	.111	3.3	3.311	.099
.70	1.021	.007	1.9	1.783	.115	3.4	3.410	.099
.75	1.028	.008	2.0	1.898	.117	3.5	3.509	.099
0.80	1.036	0.009	2.1	2.015	0.117	3.6	3.608	0.098
.85	1.045	.011	2.2	2.132	.117	3.7	3.706	.098
.90	1.057	.013	2.3	2.248	.115	3.8	3.804	.098
.95	1.070	.016	2.4	2.364	.113	3.9	3.902	.098
1.00	1.086	....	2.5	2.477	.111	4.0	4.000	....

MISCELLANEOUS FORMULAS AND DATA

78. WAVE LENGTH AND FREQUENCY OF RESONANCE

$$\lambda_{cm} = 1.8838 \times 10^{11} \sqrt{LC} \text{ (cgs electromagnetic units)} \tag{214}$$

$$= 6.283 \sqrt{L_{\text{cgs electromagnetic}} \, C_{\text{cgs electrostatic}}} \tag{215}$$

$$\lambda_m = 0.05957 \sqrt{L_{\text{cgs electromagnetic}} \, C_{\text{micromicrofarad}}} \tag{216}$$

$$= 1.884 \sqrt{L_{\text{microhenry}} \, C_{\text{micromicrofarad}}} \tag{217}$$

$$= 1884 \sqrt{L_{\text{microhenry}} \, C_{\text{microfarad}}} \tag{218}$$

$$= 59\,570 \sqrt{L_{\text{millihenry}} \, C_{\text{microfarad}}} \tag{219}$$

$$= 1\,884\,000 \sqrt{L_{\text{henry}} \, C_{\text{microfarad}}} \tag{220}$$

$$f = \frac{159.2}{\sqrt{L_{\text{henry}} \, C_{\text{microfarad}}}} \tag{221}$$

$$= \frac{5033}{\sqrt{L_{\text{millihenry}} \, C_{\text{microfarad}}}} \tag{222}$$

$$= \frac{159\,200}{\sqrt{L_{\text{microhenry}} \, C_{\text{microfarad}}}} \tag{223}$$

$$\omega = \frac{1000}{\sqrt{L_{\text{henry}} \, C_{\text{microfarad}}}} \tag{224}$$

$$= \frac{31620}{\sqrt{L_{\text{millihenry}} \, C_{\text{microfarad}}}} \tag{225}$$

$$= \frac{1\,000\,000}{\sqrt{L_{\text{microhenry}} \, C_{\text{microfarad}}}} \tag{226}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \tag{227}$$

$$\lambda_m = \frac{2.998 \times 10^8}{f} \tag{228}$$

$$= \frac{1.884 \times 10^9}{\omega} \tag{229}$$

79. MISCELLANEOUS RADIO FORMULAS

When units are not specified, international electric units are to be understood. These are the ordinary units, based on the international ohm and ampere, the centimeter and the second. Full information is given on electric units in reference No. 152, Appendix 2.

Current in Simple Series Circuit.—

$$I=\frac{E}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}} \qquad (230)$$

Phase Angle.—

$$\tan\theta=\frac{X}{R}=\frac{X_L-X_c}{R} \qquad (231)$$

$$=\frac{\omega L-\frac{1}{\omega C}}{R} \text{ in simple series circuit.} \qquad (232)$$

Sharpness of Resonance.—

$$\frac{\sqrt{\frac{I_r^2-I_1^2}{I_1^2}}}{\frac{\pm(C_r-C)}{C}}=\frac{1}{R\omega C_r}=\frac{\omega L}{R} \qquad (233)$$

(See p. 37.)

Current at Parallel Resonance.—

$$I=\frac{E\,R}{R^2+\omega^2L^2} \qquad (234)$$

(See p. 39.)

Coefficient of Coupling.—

$$k=\frac{X_m}{\sqrt{X_1X_2}} \qquad (235)$$

$$=\frac{M}{\sqrt{L_1L_2}} \text{ for direct and inductive coupling} \qquad (236)$$

$$=\frac{\sqrt{C_1C_2}}{C_m} \text{ for capacitive coupling.} \qquad (237)$$

(See p. 49.)

Power Input in Condenser—

$$P=0.5\times10^{-6}NCE_0^2 \text{ watts} \qquad (238)$$

for C in microfarads, E_0 in volts, and N = number of charges per second.

Power Loss in Condenser—

$$P = \omega C E^2 \sin \psi \tag{239}$$

Condenser Phase Difference—

$$\psi = r\omega C \tag{240}$$

for ψ in radians, r in ohms, C in farads.

$$\psi = 0.1079 \frac{rC}{\lambda} \text{ degrees} \tag{241}$$

for r in ohms, C in micromicrofarads, λ in meters.

$$\psi = 389. \frac{rC}{\lambda} \text{ seconds} \tag{242}$$

for r in ohms, C in micromicrofarads, λ in meters.

$$r = \psi \times \frac{0.001}{C} \times \frac{\lambda}{1000} \times 0.154 \text{ ohms} \tag{243}$$

for ψ in minutes, C in microfarads, λ in meters.

Energy Associated with Inductance—

$$W = \frac{1}{2} L I^2 \tag{244}$$

Inductance of Coil Having Capacity:

$$L_a = \frac{L}{1 - \omega^2 CL} \tag{245}$$

for C in farads, L in the denominator in henries.

$$L_a = L \left(1 + 3.553 \frac{CL}{\lambda^2} \right) \text{ approximately} \tag{246}$$

for λ in meters, C in micromicrofarads, L in the parentheses in microhenries. This formula is accurate when the last term is small compared with unity.

Current Transformer—

$$\frac{I_1}{I_2} = \frac{n_2}{n_1} \left(1 + \frac{aR_2}{\omega L_2} \right) \tag{247}$$

(See p. 154.)

Audibility—

$$\frac{I}{I_t} = \frac{s+t}{s} \tag{248}$$

(See p. 166.)

Natural Oscillations of Horizontal Antenna.—

$$\lambda = \frac{1199}{m}\sqrt{C_o L_o},\ m = 1, 3, 5, \ \ldots \qquad (249)$$

for λ in meters, C_0 = capacity in microfarads for uniform voltage, L_0 = inductance in microhenries for uniform current.

Approximate Wave Length of Resonance for Loaded Antenna.—

$$\lambda = 1884\sqrt{C_0\left(L + \frac{L_o}{3}\right)} \qquad (250)$$

where L = inductance of loading coil in microhenries and other quantities are as in preceding formula.

Radiation Resistance of an Antenna.—

$$R = 1580\left(\frac{h}{\lambda}\right)^2 \text{ ohms} \qquad (251)$$

where h = height from ground to center of capacity, and h and λ are in the same units, and λ is considerably greater than the fundamental wave length.

Electron Flow From Hot Filament.—

$$I_s = AT^{1/2}\epsilon^{-\frac{b}{T}} \qquad (252)$$

where I_s = electron current in milliamperes per centimeter² of filament surface, T = absolute temperature, and A and b depend on metal of filament; for tungsten $A = 2.5 \times 10^{10}$, $b = 52500$.

Electron Current in 3-Electrode Tube.—

$$I_s = k\ (E_s + k_1 v_1)^{3/2} \qquad (253)$$

where E_B = plate voltage, v_1 = grid voltage, k_1 = amplification constant.

Resistance Measurement by Resistance—Variation Method Using Undamped Emf.—

$$R = R_1\frac{I_1}{I - I_1} \qquad (254)$$

Resistance Measurement by Resistance—Variation Method Using Impulse Excitation.—

$$R = R_1\frac{I_1^2}{I^2 - I_1^2} \qquad (255)$$

Resistance Measurement by Reactance-Variation Method Using Undamped emf.—

$$R = X_1\sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \qquad (256)$$

where X_1 = change of reactance between the two observations of current. Various particular cases of this formula are given in

Natural Frequency of Simple Series Circuit.—

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{CL}-\frac{R^2}{4L^2}} \tag{257}$$

$$\omega=\frac{1}{\sqrt{CL}\sqrt{1+\left(\frac{\delta}{2\pi}\right)^2}} \tag{258}$$

Number of Oscillations to Reduce Current to 1 Per Cent of Initial Value in Wave Train.—

$$n=\frac{4.6}{\delta} \tag{259}$$

Logarithmic Decrement.—

$$\delta=\log_e\frac{I_1}{I_2}=\frac{a}{f} \tag{260}$$

$$=\pi\frac{R}{\omega L}=\pi R\omega C=\pi R\sqrt{\frac{C}{L}}$$

$$=\frac{\pi}{\text{sharpness of resonance}}$$

$=\pi\times$ phase difference of condenser or coil, the resistance being in one or the other

$$=\frac{\text{average energy dissipated per cycle}}{2\times\text{average magnetic energy at the current maxima}}$$

$$\delta=0.00167\,\frac{R\lambda}{L} \tag{261}$$

for R in ohms, λ in meters, L in microhenries.

$$\delta=5918\,\frac{RC}{\lambda} \tag{262}$$

for R in ohms, λ in meters, C in microfarads.

$$\delta=3.1416\,R\sqrt{\frac{C}{L}} \tag{263}$$

for R in ohms, C in microfarads, L in microhenries.

Current at resonance Produced by Slightly Damped emf Induced in a Circuit.—

$$I^2=\frac{N\,E_0^2}{16f^3L^2\delta'\delta\,(\delta'+\delta)} \tag{264}$$

Decrement Measurement by Reactance—Variation Method.—

$$\delta'+\delta=\pi\frac{C_2-C_1}{C_2+C_1}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{265}$$

(See p. 187 for variations of this formula.)

80. PROPERTIES OF METALS

TABLE 21

Metal	Microhm-centimeters at 20° C	Temperature coefficient at 20° C	Specific gravity	Tensile strength, lbs./in.²	Melting point, °C
Advance. See Constantan.					
Aluminum	2.828	0.0039	2.70	30 000	659
Antimony	41.7	.0036	6.6	....	630
Bismuth	120	.004	9.8	....	271
Brass	7	.002	8.6	70 000	900
Cadmium	7.6	.0038	8.6	....	321
Calido. See Nichrome.					
Climax	87	.0007	8.1	150 000	1250
Constantan	49	.00001	8.9	120 000	1190
Copper, annealed	1.7241	.00393	8.89	30 000	1083
Copper, hard-drawn	1.771	.00382	8.89	60 000	....
Eureka. See Constantan.					
Excello	92	.00016	8.9	95 000	1500
German silver, 18 per cent	33	.0004	8.4	150 000	1100
German silver, 30 per cent. See Constantan.					
Gold	2.44	.00342	19.3	20 000	1063
Ia Ia. See Constantan.					
Ideal. See Constantan.					
Iron, 99.98 per cent pure	10	.0050	7.8	....	1530
Iron. See Steel.					
Lead	22	.0039	11.4	3 000	327
Magnesium	4.6	.004	1.74	33 000	651
Manganin	44	.00001	8.4	150 000	910
Mercury	95.783	.00089	13.546	0	−38.9
Molybdenum, drawn	5.7	.004	9.0	....	2500
Monel metal	42	.0020	8.9	160 000	1300
Nichrome	100	.0004	8.2	150 000	1500
Nickel	7.8	.006	8.9	120 000	1452
Palladium	11	.0033	12.2	39 000	1550
Phosphor bronze	7.8	.0018	8.9	25 000	750
Platinum	10	.003	21.4	50 000	1755
Silver	1.59	.0038	10.5	42 000	960
Steel, E. B. B.	10.4	.005	7.7	53 000	1510
Steel, B. B.	11.9	.004	7.7	58 000	1510
Steel, Siemens-Martin	18	.003	7.7	100 000	1510
Steel, manganese	70	.001	7.5	230 000	1260
Superior. See Climax.					
Tantalum	15.5	.0031	16.6	....	2850
Therlo	47	.00001	8.2	....	....
Tin	11.5	.0042	7.3	4000	232
Tungsten, drawn	5.6	.0045	19	500 000	3000
Zinc	5.8	.0037	7.1	10 000	419

The resistivities given in Table 21 are values of ρ in the equation $R_o = \rho\frac{l}{s}$, where l = length in centimeters and s = cross section in square centimeters. This formula gives the low-frequency or direct-current resistance of a conductor. For the calculation of resistances at high frequencies, see Tables 17 to 20, pages 309–311.

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The values given for resistivity and temperature coefficient of copper are the international standard values for commercial copper. Any departure from this resistivity is accompanied by an inverse variation in the temperature coefficient. This is true in a general way for other metal elements. In the case of copper the resistivity and temperature coefficient are inversely proportional, to a high degree of accuracy.

The "temperature coefficient at 20°C" is α_{20} in the equation $R_t = R_{20}\ (1 + \alpha_{20}[t-20])$. In some cases the temperature variation does not follow a straight-line law; in such cases α_{20} applies only to a small range of temperature close to 20°. Steel is an example, the resistance rise at high temperatures being faster than proportional to temperature.

Constantan and the other wires (Advance, etc.) having substantially the same properties, are alloys of approximately 60 per cent copper and 40 per cent nickel. They are used in rheostats and measuring instruments.

German silver is an alloy of copper, nickel, and zinc. The per cent stated indicates the percentage of nickel.

Manganin contains about 84 per cent copper, 12 per cent manganese, and 4 per cent nickel. It is the usual material in resistance coils. Its very small thermal electromotive force against copper is one of its main advantages. The similar alloy, therlo, is used for the same purposes.

Monel metal is an alloy containing approximately 71 per cent nickel, 27 per cent copper, and 2 per cent iron.

NOTICE

Appendices 1 and 2, pages 319 to 333, have been purposely omitted from this printing because the material is out-of-date.

Appendix 3, "Symbols used in the Circular", has been transposed from page 334 to page 2.

An errata, listing the pages and corrections, is also printed on page 2.

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